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In order to continue to add value to our subscribers, we have put together our “HSC Comprehensive Revision Series” that we recommend motivated students aiming for a Band 5 or 6 result should **attempt, carefully review and annotate** in Term 3, creating a concise and high quality revision resource.

Note that our “Final HSC Revision Set” for interested students will be available at the start of October in the final stretch before the Mathematics HSC exam on 25 October, 2018.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create an extremely important revision set that ensures students cover a wide cross-section of the key areas we have carefully identified.

IMPORTANT: If students have been exposed to many of the questions in these worksheets during the year, we say great! In sports vernacular, this is where cobwebs are turned into cables through repetition, confidence is built and speed through the paper is developed (an aspect we regard as critical to peak achievement).

[HSC Final Study: 2UA Topics 3-5](#) (~13% historical contribution)

[Key Areas addressed by this worksheet](#)

Topic 3: Probability

- tree diagrams (notably absent in both 2016 and 2017);
- application of complementary probability, i.e. $P(E) = 1 - P(\text{complement})$; standard probability examples;
- cross topic combination with *sum to infinity* that was asked, and poorly answered in 2013 and 2016.

Topic 4: Real Functions

- graphic representation of functions; odd functions (poorly answered in 2016);
- harder example of circle equations (asked in 6 out of last 7 years, not 2017);
- a strong mark allocation in the 2017 exam.

Topic 5: Trig Ratios

- trig equations for calculating angles in radians; harder examples of trig equations within specified range that has caused issues in the past (asked 7 of last 10 years but not in 2017);
- vanilla application of sin and cosine rules that have surprisingly caused problems in the past and reviewed;
- underexamined in 2017 vs historical average;
- bearings (last asked in 2014 with a significant mark allocation).

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“Not only has SmarterMaths saved hours of my life, but the statistical analysis on every topic and the marker's comments are features we rarely find elsewhere ... truly effective!”

~ Sylvia Park, Science/Mathematics Teacher,
Girraween High School

Mathematics (Advanced)
HSC Comprehensive Revision Series



- 3. Probability
- 4. Real Functions
- 5. Trig Ratios: Exact Trig Ratios and Other Identities, Sine and Cosine Rules, Bearings

Teacher: SmarterMaths

Exam Equivalent Time: 75 minutes (based on HSC allocation of 1.5 minutes approx. per mark)

Questions

1. Probability, 2UA 2015 HSC 4 MC

The probability that Mel's soccer team wins this weekend is $\frac{5}{7}$.

The probability that Mel's rugby league team wins this weekend is $\frac{2}{3}$.

What is the probability that neither team wins this weekend?

- (A) $\frac{2}{21}$
- (B) $\frac{10}{21}$
- (C) $\frac{13}{21}$
- (D) $\frac{19}{21}$

2. Real Functions, 2UA 2013 HSC 3 MC

Which inequality defines the domain of the function $f(x) = \frac{1}{\sqrt{x+3}}$?

- (A) $x > -3$
- (B) $x \geq -3$
- (C) $x < -3$
- (D) $x \leq -3$

3. Trig Ratios, 2UA 2012 HSC 6 MC

What are the solutions of $\sqrt{3} \tan x = -1$ for $0 \leq x \leq 2\pi$?

- (A) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$
- (B) $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$
- (C) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$
- (D) $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

4. Trig Ratios, 2UA 2017 HSC 7 MC

Which expression is equivalent to $\tan \theta + \cot \theta$?

- (A) $\operatorname{cosec} \theta + \sec \theta$
- (B) $\sec \theta \operatorname{cosec} \theta$
- (C) 2
- (D) 1

5. Probability, 2UA 2014 HSC 10 MC

Three runners compete in a race. The probabilities that the three runners finish the race in under 10 seconds are $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{2}{5}$ respectively.

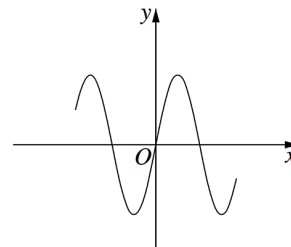
What is the probability that at least one of the three runners will finish the race in under 10 seconds?

- (A) $\frac{1}{60}$
 - (B) $\frac{37}{60}$
 - (C) $\frac{3}{8}$
 - (D) $\frac{5}{8}$
-

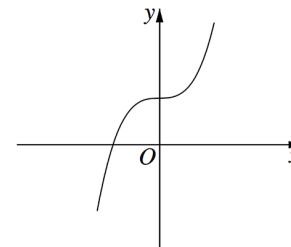
6. Real Functions, 2UA 2016 HSC 4 MC

Which diagram shows the graph of an odd function?

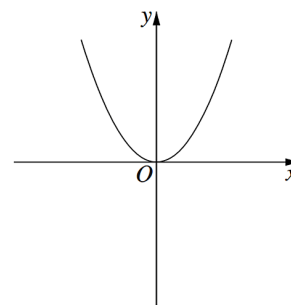
(A)



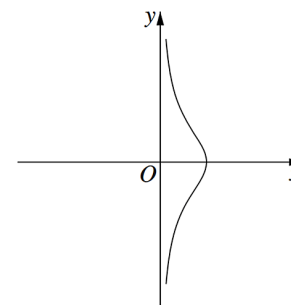
(B)



(C)

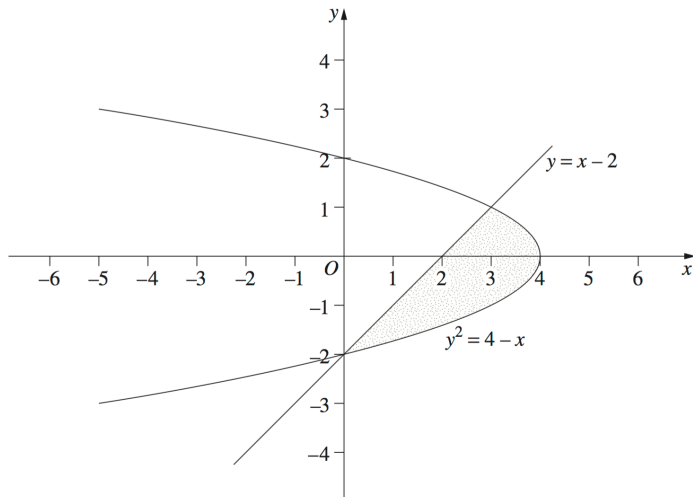


(D)



7. Real Functions, 2UA 2012 HSC 8 MC

The diagram shows the region enclosed by $y = x - 2$ and $y^2 = 4 - x$.



Which of the following pairs of inequalities describes the shaded region in the diagram?

- (A) $y^2 \leq 4 - x$ and $y \leq x - 2$
 - (B) $y^2 \leq 4 - x$ and $y \geq x - 2$
 - (C) $y^2 \geq 4 - x$ and $y \leq x - 2$
 - (D) $y^2 \geq 4 - x$ and $y \geq x - 2$
-

8. Trig Ratios, 2UA 2014 HSC 7 MC

How many solutions of the equation $(\sin x - 1)(\tan x + 2) = 0$ lie between 0 and 2π ?

- (A) 1
 - (B) 2
 - (C) 3
 - (D) 4
-

9. Probability, 2UA 2006 HSC 4c

A chessboard has 32 black squares and 32 white squares. Tanya chooses three different squares at random.

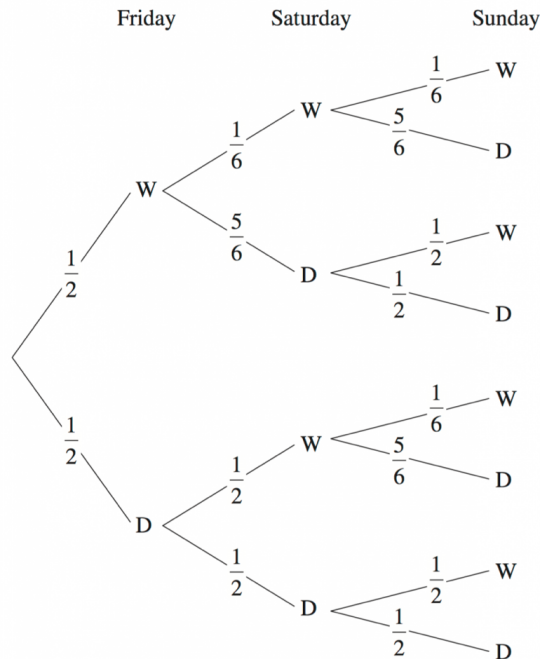
- (i) What is the probability that Tanya chooses three white squares? (2 marks)
 - (ii) What is the probability that the three squares Tanya chooses are the same colour?. (1 mark)
 - (iii) What is the probability that the three squares Tanya chooses are not the same colour? (1 mark)
-

10. Probability, 2UA 2015 HSC 14b

Weather records for a town suggest that:

- if a particular day is wet (W), the probability of the next day being dry is $\frac{5}{6}$
- if a particular day is dry (D), the probability of the next day being dry is $\frac{1}{2}$.

In a specific week Thursday is dry. The tree diagram shows the possible outcomes for the next three days: Friday, Saturday and Sunday.



- Show that the probability of Saturday being dry is $\frac{2}{3}$. (1 mark)
- What is the probability of both Saturday and Sunday being wet? (2 marks)
- What is the probability of at least one of Saturday and Sunday being dry? (1 mark)

11. Real Functions, 2UA 2010 HSC 1g

Let $f(x) = \sqrt{x - 8}$. What is the domain of $f(x)$? (1 mark)

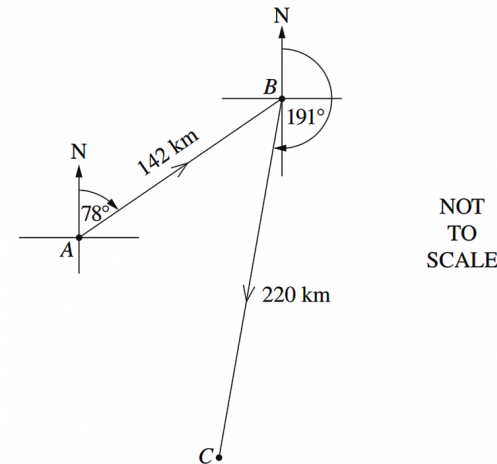
12. Trig Ratios, 2UA 2005 HSC 3b

The lengths of the sides of a triangle are 7 cm, 8 cm and 13 cm.

- Find the size of the angle opposite the longest side. (2 marks)
- Find the area of the triangle. (1 marks)

13. Trig Ratios, 2UA 2014 HSC 13d

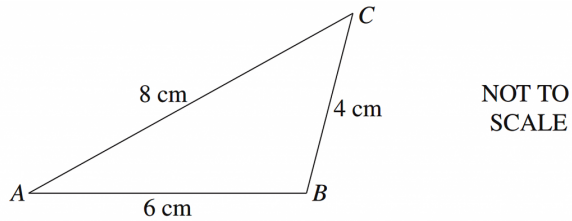
Chris leaves island A in a boat and sails 142 km on a bearing of 078° to island B . Chris then sails on a bearing of 191° for 220 km to island C , as shown in the diagram.



- Show that the distance from island C to island A is approximately 210 km. (2 marks)
- Chris wants to sail from island C directly to island A . On what bearing should Chris sail? Give your answer correct to the nearest degree. (3 marks)

14. Trig Ratios, 2UA 2015 HSC 13a

The diagram shows $\triangle ABC$ with sides $AB = 6$ cm, $BC = 4$ cm and $AC = 8$ cm.



(i) Show that

$$\cos A = \frac{7}{8}. \quad (1 \text{ mark})$$

(ii) By finding the exact value of $\sin A$, determine the exact value of the area of $\triangle ABC$. (2 marks)

15. Probability, 2UA 2007 HSC 9b

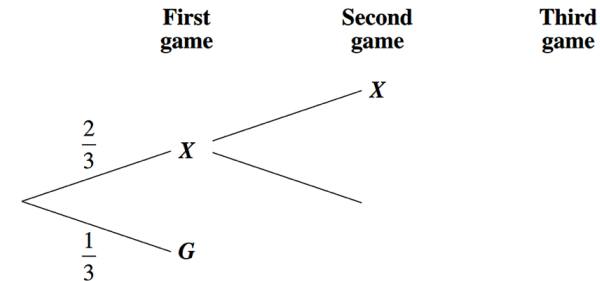
A pack of 52 cards consists of four suits with 13 cards in each suit.

- (i) One card is drawn from the pack and kept on the table. A second card is drawn and placed beside it on the table. What is the probability that the second card is from a different suit to the first? (1 mark)
- (ii) The two cards are replaced and the pack shuffled. Four cards are chosen from the pack and placed side by side on the table. What is the probability that these four cards are all from different suits?. (2 marks)

16. Probability, 2UA 2008 HSC 7c

Xena and Gabrielle compete in a series of games. The series finishes when one player has won two games. In any game, the probability that Xena wins is $\frac{2}{3}$ and the probability that Gabrielle wins is $\frac{1}{3}$.

Part of the tree diagram for this series of games is shown.



- (i) Copy and complete the tree diagram showing the possible outcomes. (1 mark)
- (ii) What is the probability that Gabrielle wins the series? (2 marks)
- (iii) What is the probability that three games are played in the series? (2 marks)

17. Trig Calculus, 2UA 2013 HSC 13a

The population of a herd of wild horses is given by

$$P(t) = 400 + 50 \cos\left(\frac{\pi}{6}t\right)$$

where t is time in months.

- (i) Find all times during the first 12 months when the population equals 375 horses. (2 marks)
- (ii) Sketch the graph of $P(t)$ for $0 \leq t \leq 12$. (2 marks)

18. Probability, 2UA 2010 HSC 8b

Two identical biased coins are tossed together, and the outcome is recorded.

After a large number of trials it is observed that the probability that both coins land showing heads is 0.36.

What is the probability that both coins land showing tails? (2 marks)

19. Real Functions, 2UA 2011 HSC 6b

A point $P(x, y)$ moves so that the sum of the squares of its distance from each of the points $A(-1, 0)$ and $B(3, 0)$ is equal to 40.

Show that the locus of $P(x, y)$ is a circle, and state its radius and centre. (3 marks)

20. Trig Ratios, 2UA 2014 HSC 15a

Find all solutions of $2 \sin^2 x + \cos x - 2 = 0$, where $0 \leq x \leq 2\pi$. (3 marks)

21. Probability, 2UA 2009 HSC 9a

Each week Van and Marie take part in a raffle at their respective workplaces.

The probability that Van wins a prize in his raffle is $\frac{1}{9}$. The probability that Marie wins a prize in her raffle is $\frac{1}{16}$.

What is the probability that, during the next three weeks, at least one of them wins a prize? (2 marks)

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Worked Solutions**1. Probability, 2UA 2015 HSC 4 MC**

$$P(\text{win at soccer}) = \frac{5}{7}$$

$$\therefore P(\text{not win at soccer}) = 1 - \frac{5}{7} = \frac{2}{7}$$

$$P(\text{win at league}) = \frac{2}{3}$$

$$\therefore P(\text{not win at league}) = \frac{1}{3}$$

$$\begin{aligned} \therefore P(\text{not win at both}) &= \frac{2}{7} \times \frac{1}{3} \\ &= \frac{2}{21} \end{aligned}$$

$\Rightarrow A$

2. Real Functions, 2UA 2013 HSC 3 MC

$$\text{Given } f(x) = \frac{1}{\sqrt{x+3}}$$

$$\begin{aligned} \text{We know } (x+3) &> 0 \\ x &> -3 \end{aligned}$$

$$\therefore \text{The domain of } f(x) \text{ is } f(x) > -3$$

$\Rightarrow A$

3. Trig Ratios, 2UA 2012 HSC 6 MC

$$\sqrt{3} \tan x = -1$$

$$\tan x = -\frac{1}{\sqrt{3}}$$

$$\text{When } \tan x = \frac{1}{\sqrt{3}}, \quad x = \frac{\pi}{6}$$

Since $\tan x$ is negative in 2nd/4th quadrant

$$\therefore x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, \dots$$

$$= \frac{5\pi}{6}, \frac{11\pi}{6}$$

$\Rightarrow D$

4. Trig Ratios, 2UA 2017 HSC 7 MC

$$\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$$

$$= \frac{1}{\cos \theta \sin \theta}$$

$$= \sec \theta \operatorname{cosec} \theta$$

$\Rightarrow B$

5. Probability, 2UA 2014 HSC 10 MC

$$P(R_1 < 10 \text{ secs}) = \frac{1}{4}$$

◆◆ Mean mark 26%

$$\Rightarrow P(R_1 \geq 10 \text{ secs}) = \frac{3}{4}$$

$$P(R_2 < 10 \text{ secs}) = \frac{1}{6}$$

$$\Rightarrow P(R_2 \geq 10 \text{ secs}) = \frac{5}{6}$$

$$P(R_3 < 10 \text{ secs}) = \frac{2}{5}$$

$$\Rightarrow P(R_3 \geq 10 \text{ secs}) = \frac{3}{5}$$

$\therefore P(\text{at least } 1 < 10 \text{ secs})$

$$= 1 - P(\text{all } \geq 10 \text{ secs})$$

$$= 1 - \frac{3}{4} \times \frac{5}{6} \times \frac{3}{5}$$

$$= 1 - \frac{45}{120}$$

$$= \frac{5}{8}$$

$\Rightarrow D$

6. Real Functions, 2UA 2016 HSC 4 MC

Odd functions occur when:

$$f(x) = -f(x)$$

◆ Mean mark 38%.

Graphically, this occurs when a function has symmetry when rotated 180° about the origin.

$\Rightarrow A$

7. Real Functions, 2UA 2012 HSC 8 MC

Using information from diagram

♦ Mean mark 44%.

(3, 0) is in the shaded region

Substituting (3,0) into $y^2 \leq 4 - x$, $0 \leq 4 - 3 \Rightarrow \text{true}$

\therefore Cannot be C or D

Similarly

(3, 0) must satisfy other inequality

i.e. $y \leq x - 2$ becomes $0 \leq 3 - 2 \Rightarrow \text{true}$

$\Rightarrow A$

8. Trig Ratios, 2UA 2014 HSC 7 MC

When $(\sin x - 1)(\tan x + 2) = 0$

$(\sin x - 1) = 0$ or $\tan x + 2 = 0$

If $\sin x - 1 = 0$

$$\sin x = 1$$

$$x = \frac{\pi}{2}, \quad 0 < x < 2\pi$$

If $\tan x + 2 = 0$

$$\tan x = -2$$

\Rightarrow Note that since $\tan \frac{\pi}{2}$ is undefined, there

are only 2 solutions when $\tan x = -2$

(which occurs in the 1st and 4th quadrants).

\therefore 2 solutions

$\Rightarrow B$

♦♦♦ Mean mark 25%, making it the toughest MC question in the 2014 exam.

COMMENT: Note that the "2 solutions" answer relies on the sum of an infinity of zeros not equalling zero. This concept created unintended difficulty in this question.

9. Probability, 2UA 2006 HSC 4c

$$\begin{aligned} \text{(i) } P(WWW) &= \frac{32}{64} \times \frac{31}{63} \times \frac{30}{62} \\ &= \frac{5}{42} \end{aligned}$$

(ii) P(same colour)

$$\begin{aligned} &= P(WWW) + P(BBB) \\ &= \frac{5}{42} + \frac{32}{64} \times \frac{31}{63} \times \frac{30}{62} \\ &= \frac{5}{42} + \frac{5}{42} \\ &= \frac{5}{21} \end{aligned}$$

(iii) P(not all the same colour)

$$\begin{aligned} &= 1 - P(\text{same colour}) \\ &= 1 - \frac{5}{21} \\ &= \frac{16}{21} \end{aligned}$$

10. Probability, 2UA 2015 HSC 14b

(i) Show $P(\text{Sat dry}) = \frac{2}{3}$

$$P(\text{Sat dry})$$

$$= P(W, D) + P(D, D)$$

$$= \left(\frac{1}{2} \times \frac{5}{6}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right)$$

$$= \frac{5}{12} + \frac{1}{4}$$

$$= \frac{2}{3} \dots \text{ as required}$$

(ii) $P(\text{Sat and Sun wet})$

$$= P(WWW) + P(DWW)$$

$$= \left(\frac{1}{2} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}\right)$$

$$= \frac{1}{72} + \frac{1}{24}$$

$$= \frac{1}{18}$$

(iii) $P(\text{At least Sat or Sun dry})$

$$= 1 - P(\text{Sat and Sun both wet})$$

$$= 1 - \frac{1}{18}$$

$$= \frac{17}{18}$$

11. Real Functions, 2UA 2010 HSC 1g

$$f(x) = \sqrt{x - 8}$$

Domain where

$$(x - 8) \geq 0$$

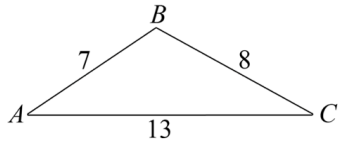
$$x \geq 8$$

♦ Mean mark 49%.

MARKER'S COMMENT: Be careful to ensure you don't incorrectly write $x > 8$. This will get zero marks!

12. Trig Ratios, 2UA 2005 HSC 3b

(i)



$\angle ABC$ is opposite the longest side

Using the cosine rule

$$\begin{aligned}\cos \angle ABC &= \frac{7^2 + 8^2 - 13^2}{2 \times 7 \times 8} \\ &= -\frac{1}{2}\end{aligned}$$

Since $\cos 60^\circ = \frac{1}{2}$ and \cos

is negative in 2nd quadrant

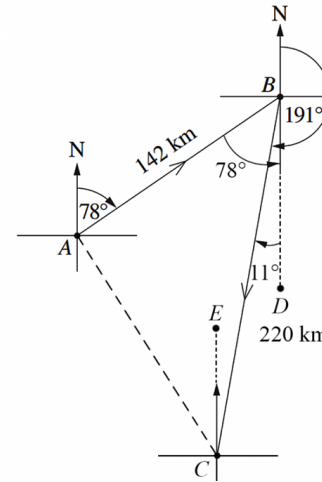
$$\begin{aligned}\angle ABC &= 180 - 60 \\ &= 120^\circ\end{aligned}$$

(ii) Using the sin rule

$$\begin{aligned}\text{Area } \triangle ABC &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 7 \times 8 \sin 120^\circ \\ &= 28 \times \frac{\sqrt{3}}{2} \\ &= 14\sqrt{3} \text{ cm}^2\end{aligned}$$

13. Trig Ratios, 2UA 2014 HSC 13d

(i)



Need to find $\angle ABC$

Let D be south of B

$$\therefore \angle CBD = 191 - 180 = 11^\circ$$

$\angle DBA = 78^\circ$ (alternate)

$$\begin{aligned}\angle ABC &= 78 - 11 \\ &= 67^\circ\end{aligned}$$

Using cosine rule

$$\begin{aligned}AC^2 &= AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos \angle ABC \\ &= 142^2 + 220^2 - 2 \times 142 \times 220 \times \cos 67^\circ \\ &= 44\,151.119\dots\end{aligned}$$

$$\therefore AC = 210.121\dots$$

$$\approx 210 \text{ km} \dots \text{ as required}$$

(ii) Need to find $\angle ACB$

Using sine rule

$$\frac{\sin \angle ACB}{142} = \frac{\sin \angle ABC}{210}$$

$$\begin{aligned}\sin \angle ACB &= \frac{142 \times \sin 67^\circ}{210} \\ &= 0.6224\dots \\ \angle ACB &= 38.494\dots \\ &= 38^\circ \text{ (nearest degree)}\end{aligned}$$

Let E be due North of C

$$\angle ECB = 11^\circ \text{ (alternate to } \angle CBD)$$

$$\begin{aligned}\therefore \angle ECA &= 38 - 11 \\ &= 27^\circ\end{aligned}$$

$$\begin{aligned}\therefore \text{Bearing of } A \text{ from } C \\ &= 360 - 27 \\ &= 333^\circ\end{aligned}$$

14. Trig Ratios, 2UA 2015 HSC 13a

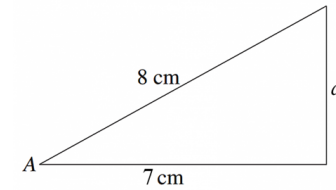
(i) Show $\cos A = \frac{7}{8}$

Using the cosine rule

$$\begin{aligned}\cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{8^2 + 6^2 - 4^2}{2 \times 8 \times 6} \\ &= \frac{64 + 36 - 16}{96} \\ &= \frac{84}{96} \\ &= \frac{7}{8} \dots \text{ as required}\end{aligned}$$

(ii)

♦ Mean mark 40%.



$$a^2 + 7^2 = 8^2$$

$$a^2 + 49 = 64$$

$$a^2 = 15$$

$$a = \sqrt{15}$$

$$\therefore \sin A = \frac{\sqrt{15}}{8}$$

$$\begin{aligned}\therefore \text{Area } \triangle ABC &= \frac{1}{2}bc \sin A \\ &= \frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{15}}{8} \\ &= 3\sqrt{15} \text{ cm}^2\end{aligned}$$

15. Probability, 2UA 2007 HSC 9b

(i) After 1st card is drawn

Cards left from another suit = 39

Cards left in pack = 51

$\therefore P$ (2nd card from the same suit)

$$= \frac{39}{51}$$

$$= \frac{13}{17}$$

(ii) P (all 4 cards from different suits)

$$= \frac{52}{52} \times \frac{39}{51} \times \frac{26}{50} \times \frac{13}{49}$$

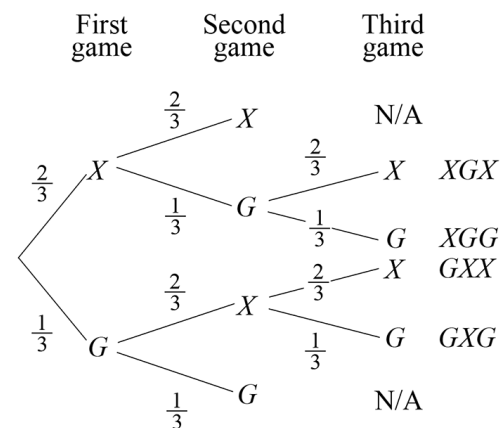
$$= \frac{2179}{20\,825}$$

$$= 0.1046\dots$$

$$= 0.105 \text{ (to 3 d.p.)}$$

16. Probability, 2UA 2008 HSC 7c

(i)



(ii) $P(G$ wins)

$$= P(XGG) + P(GXG) + P(GG)$$

$$= \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3}$$

$$= \frac{2}{27} + \frac{2}{27} + \frac{1}{9}$$

$$= \frac{7}{27}$$

(iii) P (3 games played)

$$= P(XG) + P(GX)$$

$$= \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3}$$

$$= \frac{4}{9}$$

Alternate solution

MARKER'S COMMENT: A tree diagram with 8 outcomes is incorrect (i.e. no third game is played if 1 player wins the first 2 games). If outcomes cannot occur, do not draw them on a tree diagram.

$P(3 \text{ games})$

$$\begin{aligned}
 &= 1 - [P(XX) + P(GG)] \\
 &= 1 - \left[\frac{2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{3} \right] \\
 &= 1 - \frac{5}{9} \\
 &= \frac{4}{9}
 \end{aligned}$$

17. Trig Calculus, 2UA 2013 HSC 13a

(i) $P(t) = 400 + 50 \cos\left(\frac{\pi}{6}t\right)$

Need to find t when $P(t) = 375$

$$375 = 400 + 50 \cos\left(\frac{\pi}{6}t\right)$$

$$50 \cos\left(\frac{\pi}{6}t\right) = -25$$

$$\cos\left(\frac{\pi}{6}t\right) = -\frac{1}{2}$$

Since $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$, and

\cos is negative in 2nd/3rd quadrants

$$\Rightarrow \frac{\pi}{6}t = \left(\pi - \frac{\pi}{3}\right), \left(\pi + \frac{\pi}{3}\right), \left(3\pi - \frac{\pi}{3}\right)$$

$$= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \dots$$

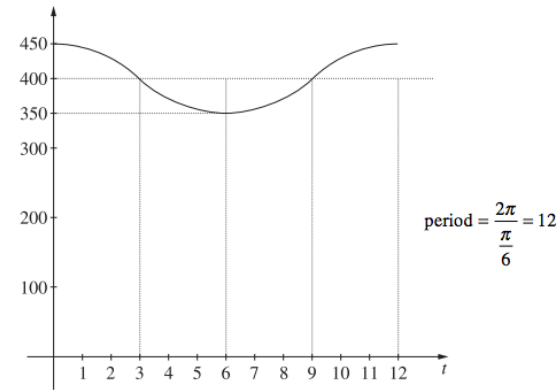
$$\therefore t = 4, 8, 16, \dots$$

\therefore In the 1st 12 months, $P(t) = 375$ when

$t = 4$ months and 8 months.

(ii)

♦ Mean mark 39%



18. Probability, 2UA 2010 HSC 8b

$$\begin{aligned}
 P(H_1 H_2) &= P(H_1) \times P(H_2) \\
 &= 0.36
 \end{aligned}$$

Since coins are identical,

$$P(H) = \sqrt{0.36}$$

$$P(H) = 0.6$$

$$\Rightarrow P(T) = 1 - P(H)$$

$$= 0.4$$

$$\therefore P(T_1 T_2) = 0.4 \times 0.4$$

$$= 0.16$$

♦♦ Mean mark 28%.

NOTE: The most common error was

$$P(T_1 T_2) = 1 - 0.36 = 0.64.$$

Ensure you understand why this does not apply.

19. Real Functions, 2UA 2011 HSC 6b

Find locus of $P(x, y)$

$$P(x, y) \quad A(-1, 0) \quad B(3, 0)$$

$$\begin{aligned} \text{Dist } PA^2 &= (y_2 - y_1)^2 + (x_2 - x_1)^2 \\ &= y^2 + (x + 1)^2 \end{aligned}$$

$$\begin{aligned} \text{Dist } PB^2 &= (y - 0)^2 + (x - 3)^2 \\ &= y^2 + (x - 3)^2 \end{aligned}$$

$$\text{Since } PA^2 + PB^2 = 40$$

$$\Rightarrow y^2 + (x + 1)^2 + y^2 + (x - 3)^2 = 40$$

$$2y^2 + x^2 + 2x + 1 + x^2 - 6x + 9 = 40$$

$$2y^2 + 2x^2 - 4x + 10 = 40$$

$$y^2 + x^2 - 2x + 5 = 20$$

$$y^2 + (x - 1)^2 + 4 = 20$$

$$y^2 + (x - 1)^2 = 16$$

$\therefore P(x, y)$ is a circle, centre is $(1, 0)$, radius 4 units.

♦ Mean mark 39%.

MARKER'S COMMENT:

Challenging question with many students unable to handle the algebra in expanding and completing the squares.

20. Trig Ratios, 2UA 2014 HSC 15a

$$2 \sin^2 x + \cos x - 2 = 0$$

$$2(1 - \cos^2 x) + \cos x - 2 = 0$$

$$2 - 2 \cos^2 x + \cos x - 2 = 0$$

$$-2 \cos^2 x + \cos x = 0$$

$$\cos x(-2 \cos x + 1) = 0$$

$$\therefore -2 \cos x + 1 = 0 \quad \text{or} \quad \cos x = 0$$

$$2 \cos x = 1 \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

Since \cos is positive in 1st/4th quadrants,

$$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$= \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\therefore x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3} \quad \text{for } 0 \leq x \leq 2\pi$$

♦ Mean mark 42%

21. Probability, 2UA 2009 HSC 9a

$$P(\text{Van loses}) = 1 - \frac{1}{9} = \frac{8}{9}$$

$$P(\text{Marie loses}) = 1 - \frac{1}{16} = \frac{15}{16}$$

$$P(\text{both lose}) = \frac{8}{9} \times \frac{15}{16} = \frac{5}{6}$$

$P(\text{At least 1 wins})$

$$= 1 - P(\text{both lose for 3 weeks})$$

$$= 1 - \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)$$

$$= 1 - \frac{125}{216}$$

$$= \frac{91}{216}$$