Thank you for subscribing to SmarterMaths Teacher Edition (Silver) in 2019.

The 2019 HSC exam is the last paper of the old course syllabus which provides a long history of high quality Mathematics exams.

In order to continue to add value to our subscribers, we have put together our "2019 HSC Comprehensive Revision Series" that we recommend motivated students aiming for a Band 5 or 6 result should ***attempt, carefully review and annotate*** in Term 3, creating a concise and high quality revision resource.

Note that our "Final HSC Revision Set" for students starting their revision later in the term will be available in early-September in the final stretch before the Mathematics HSC exam on 25 October, 2019.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create an extremely important revision set that ensures students cover a wide cross-section of the key areas we have carefully identified.

IMPORTANT: If students have been exposed to many of the questions in these worksheets during the year, we say great! In sports vernacular, this is where cobwebs are turned into cables through repetition, confidence is built and speed through the paper is developed (an aspect we regard as critical to peak achievement).

HSC Final Study: 2UA Topics 3-5 (~13% historical contribution)

Key Areas addressed by this worksheet

Topic 3: Probability

- Revision focus as past HSC cohorts have found this topic area very challenging with around half the questions examined producing sub-50% mean marks;
- Review of tree diagrams - specifically examined in 2014 and 2015, but notably absent for the last 3 years (includes the slightly different tree diagram required in *2008 Q7c*);
- Review example that is most efficiently solved using an array (sample space "table");
- Examples that require the application of complementary probability: ***P(E) = 1 - P(complement)*** - a concept that is consistently examined;
- Cross topic example requiring *sum to infinity* calculations (caused significant problems in the past).

Topic 4: Real Functions

- Review graphic representations of inequalities (most commonly asked question type) and domain restrictions in certain functions (including the poorly answered question on *odd functions* in 2016).
- Harder examples of *circle equations* – asked in *7 out of the last 9 years (including 2018).*

Topic 5: Trig Ratios

- Exact Trig Ratios and Other Identities: has been examined *at least* once each year (notably 3 times in 2016) in questions of varying difficulty, producing sub-50% mean marks in 4 out of the last 6 years;
- Review of simple trig equation that require students to answer in exact radian form within a specified range (most common question type, often poorly answered);
- Sine, Cosine Rules and Bearings: have seen an uptick in mark allocation in recent times, appearing in the last 7 HSC exams. This sub-topic was allocated a whopping 6 marks in 2018 and 3 marks in 2017;
- Vanilla examples of *apply the formula* style problems that have caused problems in the past are reviewed (note *2015 Q13a* and *2013 Q14c* both resulted in sub-50% mean marks).
- Bearings is reviewed (most recently tested in 2018 and 2014).

ADVANCED MATHEMATICS: 2019 HSC Revision Series

- T3 Probability
- T4 Real Functions
- T5 Trig Ratios

Teacher: Smarter Maths

Exam Equivalent Time: 75 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

Questions

1. Trig Ratios, 2UA 2017 HSC 7 MC

Which expression is equivalent to $\tan\theta + \cot\theta$?

(A) $\operatorname{cosec}\theta + \sec\theta$

(B) $\sec\theta \operatorname{cosec}\theta$

(C) 2

(D) 1

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2. Probability, 2UA 2014 HSC 10 MC

Three runners compete in a race. The probabilities that the three runners finish the race in under 10 seconds are $\frac{1}{4}, \frac{1}{6}$ and $\frac{2}{5}$ respectively.

What is the probability that at least one of the three runners will finish the race in under 10 seconds?

(A) $\frac{1}{60}$

(B) $\frac{37}{60}$

(C) $\frac{3}{8}$

(D) $\frac{5}{8}$

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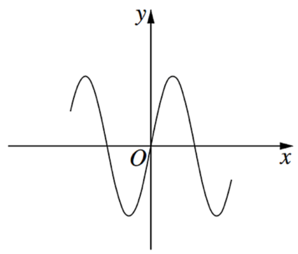
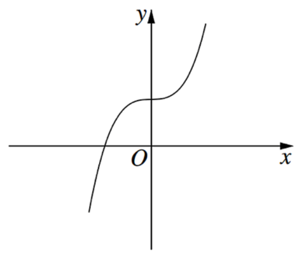
3. Real Functions, 2UA 2016 HSC 4 MC

Which diagram shows the graph of an odd function?

(A)



(B)



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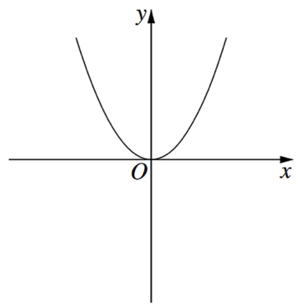

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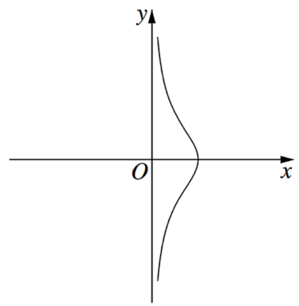

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4. Probability, 2UA 2018 HSC 6 MC

A runner has four different pairs of shoes.

If two shoes are selected at random, what is the probability that they will be a matching pair?

(A) $\frac{1}{56}$

(B) $\frac{1}{16}$

(C) $\frac{1}{7}$

(D) $\frac{1}{4}$

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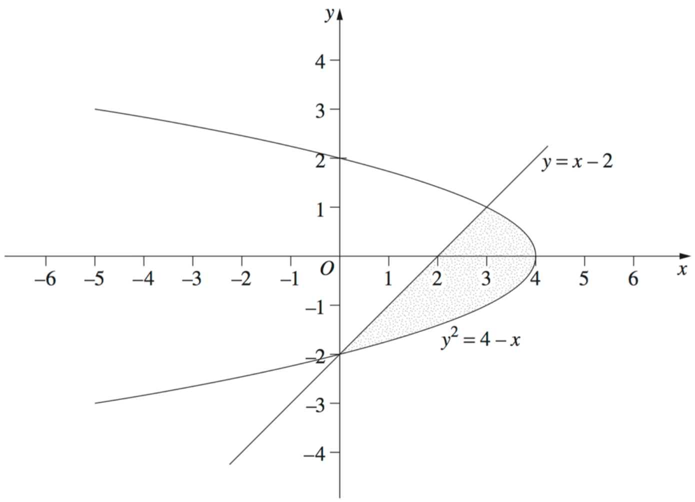
5. Real Functions, 2UA 2012 HSC 8 MC

The diagram shows the region enclosed by $y = x - 2$ and $y^2 = 4 - x$.



Which of the following pairs of inequalities describes the shaded region in the diagram?

(A) $y^2 \leq 4 - x \quad \text{and} \quad y \leq x - 2$

(B) $y^2 \leq 4 - x \quad \text{and} \quad y \geq x - 2$

(C) $y^2 \geq 4 - x \quad \text{and} \quad y \leq x - 2$

(D) $y^2 \geq 4 - x \quad \text{and} \quad y \geq x - 2$

6. Trig Ratios, 2UA 2014 HSC 7 MC

How many solutions of the equation $(\sin x - 1)(\tan x + 2) = 0$ lie between 0 and 2π?

(A) 1

(B) 2

(C) 3

(D) 4

7. Trig Ratios, 2UA 2016 HSC 8 MC

How many solutions does the equation $|\cos(2x)| = 1$ have for $0 \leq x \leq 2\pi$?

(A) 1

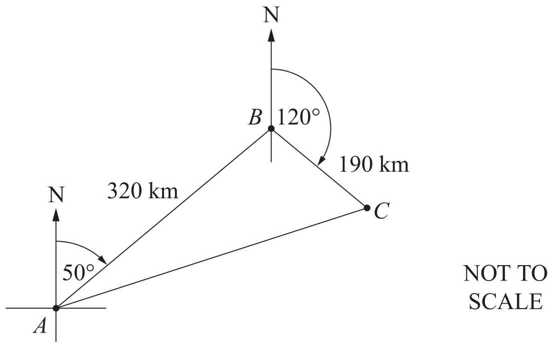
(B) 3

(C) 4

(D) 5

8. Trig Ratios, 2UA 2018 HSC 12a

A ship travels from Port *A* on a bearing of 050° for 320 km to Port *B*. It then travels on a bearing of 120° for 190 km to Port *C*.



(i) What is the size of $\angle ABC$? *(1 mark)*

(ii) What is the distance from Port *A* to Port *C* ? Answer to the nearest 10 kilometres. *(2 marks)*

9. Probability, 2UA 2010 HSC 4c

There are twelve chocolates in a box. Four of the chocolates have mint centres, four have caramel centres and four have strawberry centres. Ali randomly selects two chocolates and eats them.

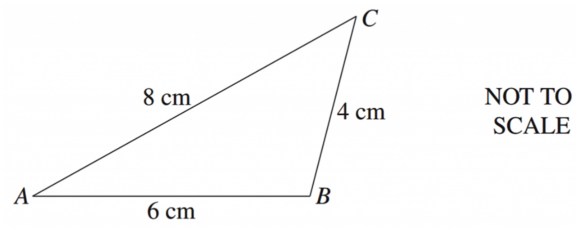
(i) What is the probability that the two chocolates have mint centres? *(1 mark)*

(ii) What is the probability that the two chocolates have the same centre? *(1 mark)*

(iii) What is the probability that the two chocolates have different centres? *(1 mark)*

10. Trig Ratios, 2UA 2015 HSC 13a

The diagram shows ΔABC with sides $AB = 6$ cm, $BC = 4$ cm and $AC = 8$ cm.



(i) Show that

$$\cos A = \frac{7}{8}.$$ *(1 mark)*

(ii) By finding the exact value of $\sin A$, determine the exact value of the area of ΔABC. *(2 marks)*

11. Probability, 2UA 2004 HSC 6c

In a game, a turn involves rolling two dice, each with faces marked 0, 1, 2, 3, 4 and 5. The score for each turn is calculated by multiplying the two numbers uppermost on the dice.

(i) What is the probability of scoring zero on the first turn? *(2 marks)*

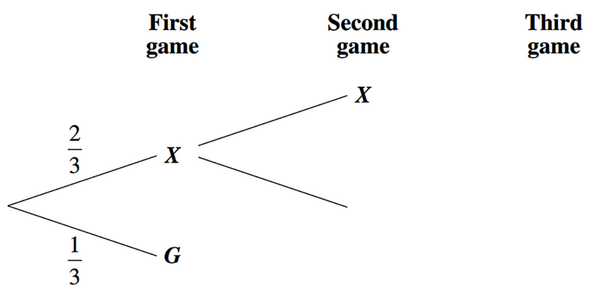
(ii) What is the probability of scoring 16 or more on the first turn? *(1 mark)*

(iii) What is the probability that the sum of the scores in the first two turns is less than 45? *(2 marks)*

12. Probability, 2UA 2008 HSC 7c

Xena and Gabrielle compete in a series of games. The series finishes when one player has won two games. In any game, the probability that Xena wins is $\frac{2}{3}$ and the probability that Gabrielle wins is $\frac{1}{3}$.

Part of the tree diagram for this series of games is shown.



(i) Copy and complete the tree diagram showing the possible outcomes. *(1 mark)*

(ii) What is the probability that Gabrielle wins the series? *(2 marks)*

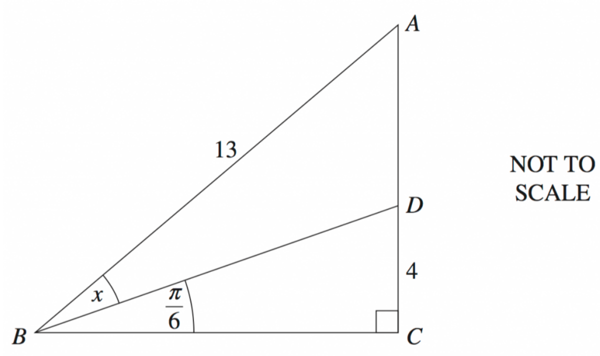
(iii) What is the probability that three games are played in the series? *(2 marks)*

13. Trig Ratios, 2UA 2004 HSC 8a

(i) Show that $\cos\theta\tan\theta = \sin\theta$. *(1 mark)*

(ii) Hence solve $8\sin\theta\cos\theta\tan\theta = \operatorname{cosec}\theta$ for $0 \leq \theta \leq 2\pi$. *(2 marks)*

14. Trig Ratios, 2UA 2013 HSC 14c



The right-angled triangle ABC has hypotenuse $AB = 13$. The point D is on AC such that $DC = 4$, $\angle DBC = \frac{\pi}{6}$ and $\angle ABD = x$.

Using the sine rule, or otherwise, find the exact value of $\sin x$. *(3 marks)*

15. Probability, 2UA 2010 HSC 8b

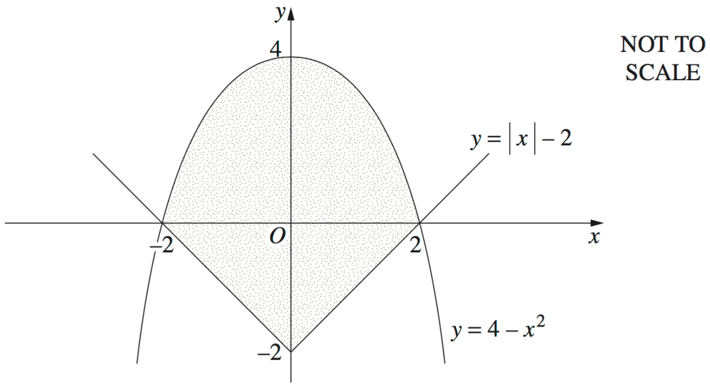
Two identical biased coins are tossed together, and the outcome is recorded.

After a large number of trials it is observed that the probability that both coins land showing heads is 0.36.

What is the probability that both coins land showing tails? *(2 marks)*

16. Real Functions, 2UA 2011 HSC 4e

The diagram shows the graphs $y = |x| - 2$ and $y = 4 - x^2$.



Write down inequalities that together describe the shaded region. *(2 marks)*

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17. Real Functions, 2UA 2011 HSC 6b

A point $P(x, y)$ moves so that the sum of the squares of its distance from each of the points $A(-1, 0)$ and $B(3, 0)$ is equal to 40.

Show that the locus of $P(x, y)$ is a circle, and state its radius and centre. *(3 marks)*

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18. Trig Ratios, 2UA 2014 HSC 15a

Find all solutions of $2\sin^2 x + \cos x - 2 = 0$, where $0 \le x \le 2\pi$. *(3 marks)*

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19. Trig Ratios, 2UA 2008 HSC 6a

Solve $2\sin^2\left(\frac{x}{3}\right) = 1$ for $-\pi \le x \le \pi$. *(3 marks)*

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20. Probability, 2UA 2013 HSC 15d

Pat and Chandra are playing a game. They take turns throwing two dice. The game is won by the first player to throw a double six. Pat starts the game.

(i) Find the probability that Pat wins the game on the first throw. *(1 mark)*

(ii) What is the probability that Pat wins the game on the first or on the second throw? *(2 marks)*

(iii) Find the probability that Pat eventually wins the game. *(2 marks)*

Worked Solutions

1. Trig Ratios, 2UA 2017 HSC 7 MC

$$\begin{aligned}\tan\theta + \cot\theta &= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\\ &= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\\ &= \frac{1}{\cos\theta\sin\theta}\\ &= \sec\theta\,\mathrm{cosec}\,\theta\end{aligned}$$

$\Rightarrow\ B$

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2. Probability, 2UA 2014 HSC 10 MC

♦♦ *Mean mark 26%*

$$\mathrm{P}(R_1 < 10\text{ secs}) = \frac{1}{4} \quad \Rightarrow \mathrm{P}\left(\overline{R}_1\right) = \frac{3}{4}$$

$$\mathrm{P}(R_2 < 10\text{ secs}) = \frac{1}{6} \quad \Rightarrow \mathrm{P}\left(\overline{R}_2\right) = \frac{5}{6}$$

$$\mathrm{P}(R_3 < 10\text{ secs}) = \frac{2}{5} \quad \Rightarrow \mathrm{P}\left(\overline{R}_3\right) = \frac{3}{5}$$

$\therefore$ P(at least 1 $<$ 10 secs)

$$\begin{aligned}&= 1 - \mathrm{P}(\text{all} \geq 10\text{ secs})\\ &= 1 - \frac{3}{4}\times\frac{5}{6}\times\frac{3}{5}\\ &= 1 - \frac{45}{120}\\ &= \frac{5}{8}\end{aligned}$$

$\Rightarrow\ D$

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3. Real Functions, 2UA 2016 HSC 4 MC

♦ *Mean mark 38%.*

Odd functions occur when:

$$f(x) = -f(x)$$

Graphically, this occurs when a function has symmetry when rotated 180° about the origin.

$\Rightarrow\ A$

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4. Probability, 2UA 2018 HSC 6 MC

♦♦ *Mean mark 31%.*

Strategy One:

Choose 1 shoe then find the probability the next choice is matching.

$$\begin{aligned}P &= 1\times\frac{1}{7}\\ &= \frac{1}{7}\end{aligned}$$

$$\begin{aligned}P &= \frac{\text{Number of desired outcomes}}{\text{Number of possibilities}}\\ &= \frac{4}{{}^8C_2}\\ &= \frac{4}{28}\\ &= \frac{1}{7}\end{aligned}$$

$\Rightarrow\ C$

Worked Solutions

5. Real Functions, 2UA 2012 HSC 8 MC

Using information from diagram

$(3, 0)$ is in the shaded region

Substituting (3,0) into $y^2 \leq 4 - x, \quad 0 \leq 4 - 3 \Rightarrow$ true

$\therefore$ Cannot be C or D

Similarly

$(3, 0)$ must satisfy other inequality

i.e. $y \leq x - 2$ becomes $0 \leq 3 - 2 \Rightarrow$ true

$\Rightarrow A$

♦ Mean mark 44%.

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6. Trig Ratios, 2UA 2014 HSC 7 MC

When $(\sin x - 1)(\tan x + 2) = 0$

$(\sin x - 1) = 0$ or $\tan x + 2 = 0$

If $\sin x - 1 = 0$

$\sin x = 1$

$x = \dfrac{\pi}{2}, \quad 0 < x < 2\pi$

If $\tan x + 2 = 0$

$\tan x = -2$

$\Rightarrow$ Note that since $\tan \dfrac{\pi}{2}$ is undefined, there are only 2 solutions when $\tan x = -2$ (which occurs in the 1st and 4th quadrants).

$\therefore$ 2 solutions

$\Rightarrow B$

♦♦♦ Mean mark 25%, making it the toughest MC question in the 2014 exam.

COMMENT: Note that the "2 solutions" answer relies on the sum of an infinity of zeros not equalling zero. This concept created unintended difficulty in this question.

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7. Trig Ratios, 2UA 2016 HSC 8 MC

$|\cos(2x)| = 1$

$\cos(2x) = \pm 1$

When $\cos(2x) = 1$

$2x = 0, 2\pi, 4\pi, \ldots$

$\therefore x = 0, \pi, 2\pi, \ldots$

When $\cos(2x) = -1$

$2x = \pi, 3\pi, 5\pi, \ldots$

$\therefore x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \ldots$

$\therefore x = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi$ for $0 \leq x \leq 2\pi$

$\Rightarrow D$

♦♦♦ Mean mark 23%.

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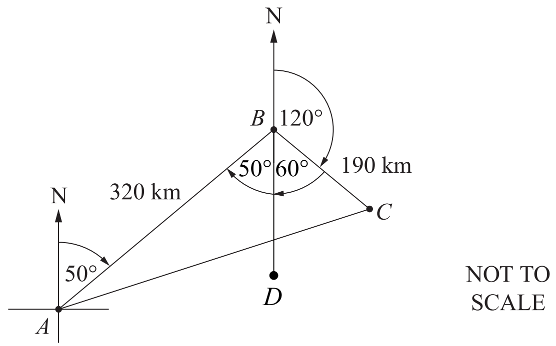
8. Trig Ratios, 2UA 2018 HSC 12a

(i)



Let D be south of B

$\angle ABD = 50^\circ$ (alternate angles)

$\angle DBC = 60^\circ$ (180° in straight line)

$\therefore \angle ABC = 50 + 60$

$= 110^\circ$

(ii) Using the cosine rule:

$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos\angle ABC$$

$$= 320^2 + 190^2 - 2 \times 320 \times 190 \times \cos 110^\circ$$

$$= 180\,089.64\ldots$$

$$\therefore AC = 424.36\ldots$$

$= 420$ km (nearest 10 km)

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9. Probability, 2UA 2010 HSC 4c

(i) $P(\text{2 mint}) = P(M_1) \times P(M_2)$

$$= \frac{4}{12} \times \frac{3}{11}$$

$$= \frac{1}{11}$$

(ii) $P(\text{2 same}) = P(M_1 M_2) + P(C_1 C_2) + P(S_1 S_2)$

$$= \frac{1}{11} + \left(\frac{4}{12} \times \frac{3}{11}\right) + \left(\frac{4}{12} \times \frac{3}{11}\right)$$

$$= \frac{3}{11}$$

(iii) Solution 1

EXAM TIP: Using $1 - P(\text{2 things the same})$ is a quicker and easier strategy here.

$P(\text{2 diff}) = 1 - P(\text{2 same})$

$$= 1 - \frac{3}{11}$$

$$= \frac{8}{11}$$

Solution 2

$P(\text{2 diff}) = P(M_1, \text{not } M_2) + P(C_1, \text{not } C_2) + P(S_1, \text{not } S_2)$

$$= \left(\frac{4}{12} \times \frac{8}{11}\right) + \left(\frac{4}{12} \times \frac{8}{11}\right) + \left(\frac{4}{12} \times \frac{8}{11}\right)$$

$$= \frac{32}{121} + \frac{32}{121} + \frac{32}{121}$$

$$= \frac{8}{11}$$

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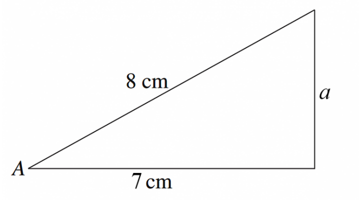
10. Trig Ratios, 2UA 2015 HSC 13a

(i) Show $\cos A = \dfrac{7}{8}$

Using the cosine rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{8^2 + 6^2 - 4^2}{2 \times 8 \times 6}$$
$$= \frac{64 + 36 - 16}{96}$$
$$= \frac{84}{96}$$
$$= \frac{7}{8} \ \ldots \ \text{as required}$$

(ii)



♦ *Mean mark 40%.*

$$a^2 + 7^2 = 8^2$$
$$a^2 + 49 = 64$$
$$a^2 = 15$$
$$a = \sqrt{15}$$
$$\therefore \ \sin A = \frac{\sqrt{15}}{8}$$

$$\therefore \ \text{Area } \Delta ABC = \frac{1}{2} bc \sin A$$
$$= \frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{15}}{8}$$
$$= 3\sqrt{15} \ \text{cm}^2$$

11. Probability, 2UA 2004 HSC 6c

(i)

Die 2 \ Die 1	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25

MARKER'S COMMENT: Students who drew up the table for the sample space were "overwhelmingly" more successful in all parts of this question.

$$\therefore P(0) = \frac{11}{36}$$

(ii) $P(\geq 16) = \frac{4}{36} = \frac{1}{9}$

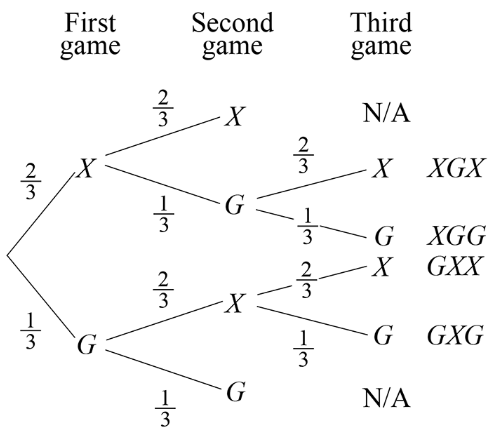
(iii) $P(\text{Sum} < 45) = 1 - P(\text{Sum} \geq 45)$

$$P(\text{Sum} \geq 45) = P(20, 25) + P(25, 20) + P(25, 25)$$
$$= \left(\frac{2}{36} \times \frac{1}{36}\right) + \left(\frac{2}{36} \times \frac{1}{36}\right) + \left(\frac{1}{36} \times \frac{1}{36}\right)$$
$$= \frac{2}{1296} + \frac{2}{1296} + \frac{1}{1296}$$
$$= \frac{5}{1296}$$

$$\therefore P(\text{Sum} < 45) = 1 - \frac{5}{1296}$$
$$= \frac{1291}{1296}$$

12. Probability, 2UA 2008 HSC 7c

(i)



MARKER'S COMMENT: A tree diagram with 8 outcomes is incorrect (i.e. no third game is played if 1 player wins the first 2 games). If outcomes cannot occur, do not draw them on a tree diagram.

(ii) $P(G \text{ wins})$

$$= P(XGG) + P(GXG) + P(GG)$$
$$= \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3}$$
$$= \frac{2}{27} + \frac{2}{27} + \frac{1}{9}$$
$$= \frac{7}{27}$$

(iii) $P(3 \text{ games played})$

$$= P(XG) + P(GX)$$
$$= \frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3}$$
$$= \frac{4}{9}$$

Alternate solution

$P(3 \text{ games})$

$$= 1 - [P(XX) + P(GG)]$$
$$= 1 - \left[\frac{2}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{1}{3}\right]$$
$$= 1 - \frac{5}{9}$$
$$= \frac{4}{9}$$

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13. Trig Ratios, 2UA 2004 HSC 8a

(i) Prove $\cos\theta\tan\theta = \sin\theta$

$$\text{LHS} = \cos\theta\tan\theta$$
$$= \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)$$
$$= \sin\theta$$
$$= \text{RHS}$$

(ii) $8\sin\theta\cos\theta\tan\theta = \operatorname{cosec}\theta$

$$\therefore 8\sin\theta(\sin\theta) = \operatorname{cosec}\theta, \quad \text{(part (i))}$$
$$8\sin^2\theta = \frac{1}{\sin\theta}$$
$$8\sin^3\theta = 1$$
$$\sin^3\theta = \frac{1}{8}$$
$$\sin\theta = \frac{1}{2}$$
$$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}. \quad (\text{for } 0 \le \theta \le 2\pi)$$

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14. Trig Ratios, 2UA 2013 HSC 14c

Find $\angle ADB$

$$\angle ADB = \frac{\pi}{6} + \frac{\pi}{2} \quad \text{(exterior angle of } \Delta BDC\text{)}$$
$$= \frac{2\pi}{3} \text{ radians}$$

Find AD

$$\tan\left(\frac{\pi}{6}\right) = \frac{4}{BC}$$
$$\frac{1}{\sqrt{3}} = \frac{4}{BC}$$
$$BC = 4\sqrt{3}$$

Using Pythagoras:

$$AC^2 + BC^2 = AB^2$$
$$AC^2 + \left(4\sqrt{3}\right)^2 = 13^2$$
$$AC^2 = 169 - 48$$
$$= 121$$
$$\Rightarrow AC = 11$$
$$\therefore AD = AC - DC$$
$$= 11 - 4$$
$$= 7$$

Using sine rule:

$$\frac{AB}{\sin\angle BDA} = \frac{AD}{\sin x}$$
$$\frac{13}{\sin\left(\frac{2\pi}{3}\right)} = \frac{7}{\sin x}$$
$$13 \times \sin x = 7 \times \sin\left(\frac{2\pi}{3}\right)$$
$$\sin x = \frac{7}{13} \times \sin\left(\frac{2\pi}{3}\right)$$
$$= \frac{7}{13} \times \frac{\sqrt{3}}{2}$$

♦ *Mean mark 36%.*
STRATEGY TIP: The hint to use the sine rule should flag to students that they will be dealing in non-right angled trig (i.e. ΔABD) and to direct their energies at initially finding $\angle ADB$ and AD.

$$= \frac{7\sqrt{3}}{26}$$

$\therefore$ The exact value of $\sin x = \dfrac{7\sqrt{3}}{26}$.

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15. Probability, 2UA 2010 HSC 8b

$$P(H_1H_2) = P(H_1) \times P(H_2)$$
$$= 0.36$$

Since coins are identical,

$$P(H) = \sqrt{0.36}$$
$$P(H) = 0.6$$
$$\Rightarrow P(T) = 1 - P(H)$$
$$= 0.4$$
$$\therefore P(T_1T_2) = 0.4 \times 0.4$$
$$= 0.16$$

♦♦ *Mean mark 28%.*
NOTE: The most common error was $P(T_1T_2) = 1 - 0.36 = 0.64$. Ensure you understand why this does not apply.

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16. Real Functions, 2UA 2011 HSC 4e

Inequalities are

$$y \leq 4 - x^2$$
$$y \geq |x| - 2$$

♦ Mean mark 46%.

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17. Real Functions, 2UA 2011 HSC 6b

♦ *Mean mark 39%.*
MARKER'S COMMENT: Challenging question with many students unable to handle the algebra in expanding and completing the squares.

Find locus of $P(x, y)$

$P(x, y) \quad A(-1,0) \quad B(3, 0)$

$$\text{Dist } PA^2 = (y_2 - y_1)^2 + (x_2 - x_1)^2$$
$$= y^2 + (x+1)^2$$
$$\text{Dist } PB^2 = (y - 0)^2 + (x - 3)^2$$
$$= y^2 + (x-3)^2$$

Since $PA^2 + PB^2 = 40$

$$\Rightarrow y^2 + (x+1)^2 + y^2 + (x-3)^2 = 40$$
$$2y^2 + x^2 + 2x + 1 + x^2 - 6x + 9 = 40$$
$$2y^2 + 2x^2 - 4x + 10 = 40$$
$$y^2 + x^2 - 2x + 5 = 20$$
$$y^2 + (x-1)^2 + 4 = 20$$
$$y^2 + (x-1)^2 = 16$$

$\therefore$ $P(x, y)$ is a circle, centre $(1, 0)$, radius 4 units.

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18. Trig Ratios, 2UA 2014 HSC 15a

♦ *Mean mark 42%*

$$2\sin^2 x + \cos x - 2 = 0$$
$$2\left(1 - \cos^2 x\right) + \cos x - 2 = 0$$
$$2 - 2\cos^2 x + \cos x - 2 = 0$$
$$-2\cos^2 x + \cos x = 0$$
$$\cos x(-2\cos x + 1) = 0$$

$$\therefore -2\cos x + 1 = 0 \quad \text{or} \quad \cos x = 0$$
$$2\cos x = 1 \qquad\qquad x = \frac{\pi}{2}, \frac{3\pi}{2}$$
$$\cos x = \frac{1}{2}$$
$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

Since cos is positive in 1st/4th quadrants,

$$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$
$$= \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\therefore x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3} \text{ for } 0 \le x \le 2\pi$$

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19. Trig Ratios, 2UA 2008 HSC 6a

$$2\sin^2\left(\frac{x}{3}\right) = 1 \text{ for } -\pi \le x \le \pi$$

$$\sin^2\left(\frac{x}{3}\right) = \frac{1}{2}$$

$$\sin\left(\frac{x}{3}\right) = \pm\frac{1}{\sqrt{2}}$$

♦♦ Although exact data not available, markers specifically mentioned this question was poorly answered.

When $\sin\left(\frac{x}{3}\right) = \frac{1}{\sqrt{2}}$

$$\frac{x}{3} = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$x = \frac{3\pi}{4}, \frac{9\pi}{4}$$

MARKER'S COMMENT: Many students had problems adjusting their answer to the given domain, especially when dealing with negative angles.

When $\sin\left(\frac{x}{3}\right) = -\frac{1}{\sqrt{2}}$

$$\frac{x}{3} = -\frac{\pi}{4}, -\frac{3\pi}{4}$$

$$x = -\frac{3\pi}{4}, -\frac{9\pi}{4}$$

$$\therefore x = -\frac{3\pi}{4} \text{ or } \frac{3\pi}{4} \text{ for } -\pi \le x \le \pi$$

20. Probability, 2UA 2013 HSC 15d

(i) P (Pat wins on 1st throw) $= P(W)$

$$P(W) = P \text{ (Pat throws 2 sixes)}$$

$$= \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{36}$$

(ii) Let $P(L) = P(\text{loss for either player on a throw}) = \frac{35}{36}$

P(Pat wins on 1st or 2nd throw)

$$= P(W) + P(LLW)$$

$$= \frac{1}{36} + \left(\frac{35}{36}\right) \times \left(\frac{35}{36}\right) \times \left(\frac{1}{36}\right)$$

$$= \frac{2521}{46\,656}$$

$$= 0.054 \quad \text{(to 3 d.p.)}$$

♦♦ Mean mark 33%
MARKER'S COMMENT: Many students did not account for Chandra having to lose when Pat wins on the ***2nd*** attempt.

(iii) P (Pat wins eventually)

$$= P(W) + P(LL\,W) + P(LL\,LL\,W) + \ldots$$

$$= \frac{1}{36} + \left(\frac{35}{36}\right)^2\left(\frac{1}{36}\right) + \left(\frac{35}{36}\right)^2\left(\frac{35}{36}\right)^2\left(\frac{1}{36}\right) + \ldots$$

$$\Rightarrow \text{ GP where } a = \frac{1}{36}, \quad r = \left(\frac{35}{36}\right)^2 = \frac{1225}{1296}$$

Since $|\,r\,| < 1$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{\frac{1}{36}}{1 - \left(\frac{1225}{1296}\right)}$$

$$= \frac{1}{36} \times \frac{1296}{71}$$

$$= \frac{36}{71}$$

$$\therefore \text{ Pat's chances to win eventually are } \frac{36}{71}.$$

♦♦♦ Mean mark 8%!
COMMENT: Be aware that diminishing probabilities and S_∞ within the Series and Applications are a natural cross-topic combination.