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Thank you for subscribing to SmarterMaths Teacher Edition (Silver) in 2019.

The 2019 paper for the HSC Mathematics course is its swansong after four illustrious decades where the syllabus has remained intact.

This is our "Final Stretch HSC Revision Series" that we recommend motivated students aiming for a Band 5 or 6 result should ***attempt, carefully review and annotate*** to create a concise and high quality revision resource that they can refer back to.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create an extremely important revision set that ensures students cover a wide cross-section of the key areas we have carefully identified.

IMPORTANT: If students have been exposed to some of the questions in these worksheets during the year, all the better. Do not underestimate the crucial skill of developing speed through the exam and few revision practices are more effective in achieving this than revisiting quality questions for a second (and third) time.

HSC Final Study: 2UA Topics 3-5 (~13% historical contribution)

Key Areas addressed by this worksheet

Topic 3: Probability

- Revision focus as past HSC cohorts have found this topic area very challenging with around half the questions examined producing sub-50% mean marks;
- Review of tree diagrams - specifically examined in 2014 and 2015, but notably absent for the last 3 years (includes the slightly different tree diagram required in *2008 Q7c*);
- Examples that require the application of complementary probability: ***P(E) = 1 - P(complement)*** - a concept that is consistently examined;

Topic 4: Real Functions

- Review graphic representations of inequalities (most commonly asked question type) and odd functions.
- Harder examples of *circle equations* – asked in *7 out of the last 9 years (including 2018).*

Topic 5: Trig Ratios

- Exact Trig Ratios and Other Identities: has been examined *at least* once each year (notably 3 times in 2016) in questions of varying difficulty, producing sub-50% mean marks in 4 out of the last 6 years;
- Review of simple trig equation that require students to answer in exact radian form within a specified range (most common question type, often poorly answered);
- Sine, Cosine Rules and Bearings: have seen an uptick in mark allocation in recent times, appearing in the last 7 HSC exams. This sub-topic was allocated a whopping 6 marks in 2018 and 3 marks in 2017;
- Vanilla examples of *apply the formula* style problems that have caused problems in the past are reviewed (*2013 Q14c* reviewed).
- Bearings is reviewed.

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"Student demand for the worksheets has quickly turned SmarterMaths into an essential part of our senior maths program."

~ Carolyn Nolan, Head Teacher of Mathematics, Lambton High

MATHEMATICS

2019 HSC Final Stretch Revision Series

- T3. Probability
- T4. Real Functions
- T5. Trig Ratios

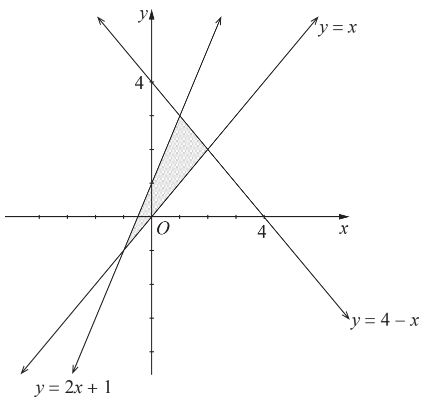
Teacher: Smarter Maths

Exam Equivalent Time: 45 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

Questions

1. Real Functions, 2UA 2017 HSC 8 MC

The region enclosed by $y = 4 - x$, $y = x$ and $y = 2x + 1$ is shaded in the diagram below.



Which of the following defines the shaded region?

(A) $y \le 2x + 1, \quad y \le 4 - x, \quad y \ge x$

(B) $y \ge 2x + 1, \quad y \le 4 - x, \quad y \ge x$

(C) $y \le 2x + 1, \quad y \ge 4 - x, \quad y \ge x$

(D) $y \ge 2x + 1, \quad y \ge 4 - x, \quad y \ge x$

2. Trig Ratios, 2UA 2012 HSC 6 MC

What are the solutions of $\sqrt{3}\tan x = -1$ for $0 \le x \le 2\pi$?

(A) $\dfrac{2\pi}{3}$ and $\dfrac{4\pi}{3}$

(B) $\dfrac{2\pi}{3}$ and $\dfrac{5\pi}{3}$

(C) $\dfrac{5\pi}{6}$ and $\dfrac{7\pi}{6}$

(D) $\dfrac{5\pi}{6}$ and $\dfrac{11\pi}{6}$

3. Trig Ratios, 2UA 2017 HSC 7 MC

Which expression is equivalent to $\tan\theta + \cot\theta$?

(A) $\operatorname{cosec}\theta + \sec\theta$

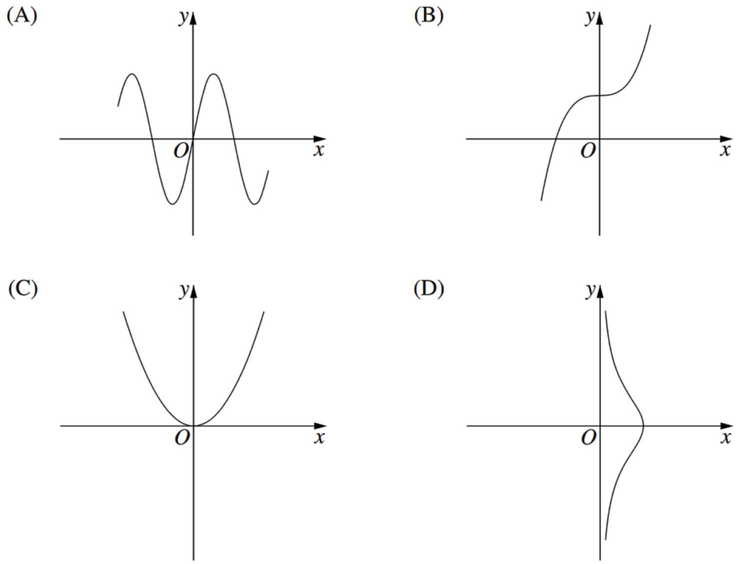
(B) $\sec\theta \operatorname{cosec}\theta$

(C) 2

(D) 1

4. Real Functions, 2UA 2016 HSC 4 MC

Which diagram shows the graph of an odd function?



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5. Probability, 2UA 2018 HSC 6 MC

A runner has four different pairs of shoes.

If two shoes are selected at random, what is the probability that they will be a matching pair?

(A) $\dfrac{1}{56}$

(B) $\dfrac{1}{16}$

(C) $\dfrac{1}{7}$

(D) $\dfrac{1}{4}$

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6. Trig Ratios, 2UA 2014 HSC 7 MC

How many solutions of the equation $(\sin x - 1)(\tan x + 2) = 0$ lie between 0 and 2π?

(A) 1

(B) 2

(C) 3

(D) 4

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7. Real Functions, 2UA 2010 HSC 1c

Write down the equation of the circle with centre (-1, 2) and radius 5. *(1 mark)*

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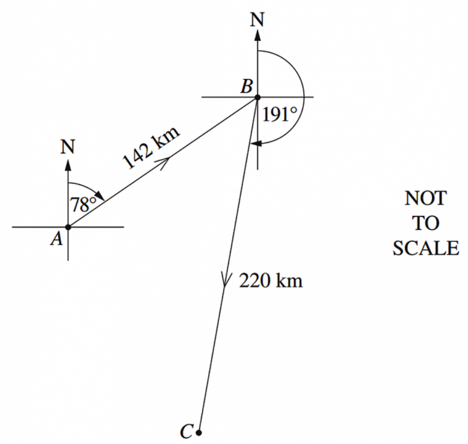
8. Trig Ratios, 2UA 2014 HSC 13d

Chris leaves island A in a boat and sails 142 km on a bearing of 078° to island B. Chris then sails on a bearing of 191° for 220 km to island C, as shown in the diagram.



(i) Show that the distance from island C to island A is approximately 210 km. *(2 marks)*

(ii) Chris wants to sail from island C directly to island A. On what bearing should Chris sail? Give your answer correct to the nearest degree. *(3 marks)*

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9. Probability, 2UA 2018 HSC 14e

Two machines, A and B, produce pens. It is known that 10% of the pens produced by machine A are faulty and that 5% of the pens produced by machine B are faulty.

(i) One pen is chosen at random from each machine.
What is the probability that at least one of the pens is faulty? *(1 mark)*

(ii) A coin is tossed to select one of the two machines. Two pens are chosen at random from the selected machine.
What is the probability that neither pen is faulty? *(2 marks)*

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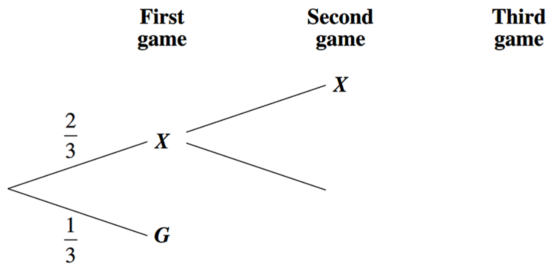
10. Probability, 2UA 2008 HSC 7c

Xena and Gabrielle compete in a series of games. The series finishes when one player has won two games. In any game, the probability that Xena wins is $\frac{2}{3}$ and the probability that Gabrielle wins is $\frac{1}{3}$.

Part of the tree diagram for this series of games is shown.



(i) Copy and complete the tree diagram showing the possible outcomes. *(1 mark)*

(ii) What is the probability that Gabrielle wins the series? *(2 marks)*

(iii) What is the probability that three games are played in the series? *(2 marks)*

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11. Trig Calculus, 2UA 2013 HSC 13a

The population of a herd of wild horses is given by

$$P(t) = 400 + 50\cos\left(\frac{\pi}{6}t\right)$$

where t is time in months.

(i) Find all times during the first 12 months when the population equals 375 horses. *(2 marks)*

(ii) Sketch the graph of $P(t)$ for $0 \leq t \leq 12$. *(2 marks)*

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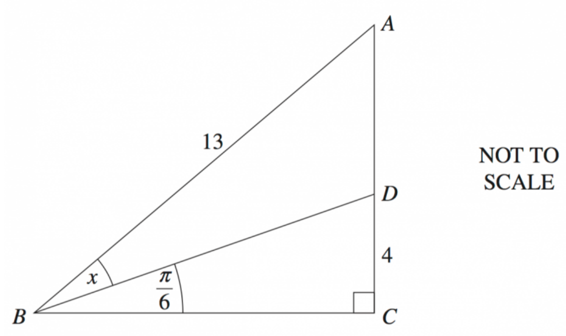
12. Trig Ratios, 2UA 2013 HSC 14c



The right-angled triangle ABC has hypotenuse $AB = 13$. The point D is on AC such that $DC = 4$, $\angle DBC = \frac{\pi}{6}$ and $\angle ABD = x$.

Using the sine rule, or otherwise, find the exact value of $\sin x$. *(3 marks)*

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13. Real Functions, 2UA 2011 HSC 6b

A point $P(x, y)$ moves so that the sum of the squares of its distance from each of the points $A(-1, 0)$ and $B(3, 0)$ is equal to 40.

Show that the locus of $P(x, y)$ is a circle, and state its radius and centre. *(3 marks)*

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Worked Solutions

Worked Solutions

1. Real Functions, 2UA 2017 HSC 8 MC

Consider $y = 2x + 1$,

Shading is below graph

$\Rightarrow y \leq 2x + 1$

Consider $y = 4 - x$,

Shading is below graph

$\Rightarrow y \leq 4 - x$

$\Rightarrow A$

2. Trig Ratios, 2UA 2012 HSC 6 MC

$$\sqrt{3}\tan x = -1$$

$$\tan x = -\frac{1}{\sqrt{3}}$$

When $\tan x = \dfrac{1}{\sqrt{3}}$, $x = \dfrac{\pi}{6}$

Since $\tan x$ is negative in $2^{\text{nd}}/4^{\text{th}}$ quadrant

$$\therefore x = \pi - \frac{\pi}{6},\ 2\pi - \frac{\pi}{6},\ \ldots$$

$$= \frac{5\pi}{6},\ \frac{11\pi}{6}$$

$\Rightarrow D$

3. Trig Ratios, 2UA 2017 HSC 7 MC

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta}$$

$$= \sec\theta\,\text{cosec}\,\theta$$

$\Rightarrow B$

4. Real Functions, 2UA 2016 HSC 4 MC

Odd functions occur when:

$$f(x) = -f(x)$$

♦ *Mean mark 38%.*

Graphically, this occurs when a function has symmetry when rotated 180° about the origin.

$\Rightarrow A$

5. Probability, 2UA 2018 HSC 6 MC

Strategy One:

Choose 1 shoe then find the probability the next choice is matching.

♦♦ *Mean mark 31%.*

$$P = 1 \times \frac{1}{7}$$

$$= \frac{1}{7}$$

$$P = \frac{\text{Number of desired outcomes}}{\text{Number of possibilities}}$$

$$= \frac{4}{{}^{8}C_2}$$

$$= \frac{4}{28}$$

$$= \frac{1}{7}$$

$\Rightarrow C$

6. Trig Ratios, 2UA 2014 HSC 7 MC

When $(\sin x - 1)(\tan x + 2) = 0$

$(\sin x - 1) = 0$ or $\tan x + 2 = 0$

If $\sin x - 1 = 0$

$\sin x = 1$

$x = \dfrac{\pi}{2}, \quad 0 < x < 2\pi$

If $\tan x + 2 = 0$

$\tan x = -2$

$\Rightarrow$ Note that since $\tan \dfrac{\pi}{2}$ is undefined, there are only 2 solutions when $\tan x = -2$ (which occurs in the 1st and 4th quadrants).

$\therefore$ 2 solutions

$\Rightarrow$ B

♦♦♦ Mean mark 25%, making it the toughest MC question in the 2014 exam.

COMMENT: Note that the "2 solutions" answer relies on the sum of an infinity of zeros not equalling zero. This concept created unintended difficulty in this question.

7. Real Functions, 2UA 2010 HSC 1c

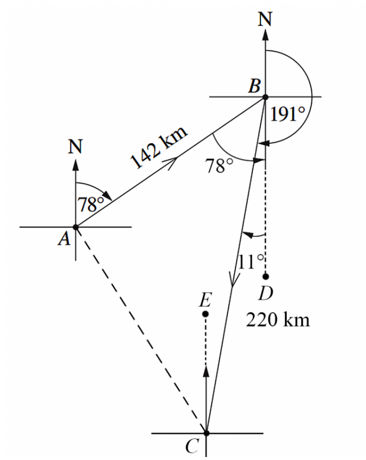
Circle with centre $(-1, 2)$, $r = 5$

$(x + 1)^2 + (y - 2)^2 = 25$

MARKER'S COMMENT: Expanding this equation is not necessarily!

8. Trig Ratios, 2UA 2014 HSC 13d

(i)



Find $\angle ABC$

Let D be south of B

$\therefore \angle CBD = 191 - 180 = 11°$

$\angle DBA = 78°$ (alternate)

$\angle ABC = 78 - 11$

$= 67°$

Using cosine rule:

$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos\angle ABC$

$= 142^2 + 220^2 - 2 \times 142 \times 220 \times \cos 67°$

$= 44\,151.119...$

$\therefore AC = 210.121...$

≈ 210 km ... as required

(ii) Find $\angle ACB$

Using sine rule:

$$\frac{\sin\angle ACB}{142} = \frac{\sin\angle ABC}{210}$$

$$\sin\angle ACB = \frac{142 \times \sin 67°}{210}$$

$$= 0.6224...$$
$$\angle ACB = 38.494...$$
$$= 38^\circ \text{ (nearest degree)}$$

Let E be due North of C

$\angle ECB = 11^\circ$ (alternate to $\angle CBD$)

$$\therefore\ \angle ECA = 38 - 11$$
$$= 27^\circ$$

$\therefore$ Bearing of A from C

$$= 360 - 27$$
$$= 333^\circ$$

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9. Probability, 2UA 2018 HSC 14e

(i) $\text{P(at least 1 faulty)} = 1 - \text{P(both faulty)}$

$$= 1 - 0.9 \times 0.95$$
$$= 1 - 0.855$$
$$= 0.145$$

(ii) P(2 non-faulty pens)

$$= \text{(choose A, NF, NF)} + P\text{(choose B, NF, NF)}$$
$$= \frac{1}{2} \times 0.9 \times 0.9 + \frac{1}{2} \times 0.95 \times 0.95$$
$$= 0.405 + 0.45125$$
$$= 0.85625$$

♦ *Mean mark 48%.*

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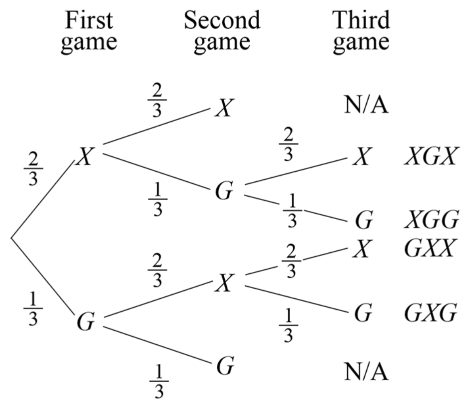
10. Probability, 2UA 2008 HSC 7c

(i)



(ii) $P(G \text{ wins})$

$$= P(XGG) + P(GXG) + P(GG)$$
$$= \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3}$$
$$= \frac{2}{27} + \frac{2}{27} + \frac{1}{9}$$
$$= \frac{7}{27}$$

MARKER'S COMMENT: A tree diagram with 8 outcomes is incorrect (i.e. no third game is played if 1 player wins the first 2 games). If outcomes cannot occur, do not draw them on a tree diagram.

(iii) $P(3 \text{ games played})$

$$= P(XG) + P(GX)$$
$$= \frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3}$$
$$= \frac{4}{9}$$

Alternate solution

$P(3 \text{ games})$

$$= 1 - [P(XX) + P(GG)]$$
$$= 1 - \left[\frac{2}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{1}{3}\right]$$

$$= 1 - \frac{5}{9}$$

$$= \frac{4}{9}$$

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11. Trig Calculus, 2UA 2013 HSC 13a

(i) $P(t) = 400 + 50\cos\left(\frac{\pi}{6}t\right)$

Need to find t when $P(t) = 375$

$$375 = 400 + 50\cos\left(\frac{\pi}{6}t\right)$$

$$50\cos\left(\frac{\pi}{6}t\right) = -25$$

$$\cos\left(\frac{\pi}{6}t\right) = -\frac{1}{2}$$

Since $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$, and cos is negative in $2^{nd}/3^{rd}$ quadrants,

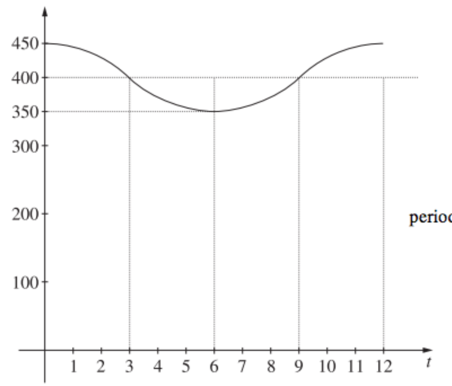
$$\Rightarrow \frac{\pi}{6}t = \left(\pi - \frac{\pi}{3}\right), \left(\pi + \frac{\pi}{3}\right), \left(3\pi - \frac{\pi}{3}\right)$$

$$= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \ldots$$

$$\therefore t = 4, 8, 16, \ldots$$

$\therefore$ In the 1st 12 months, $P(t) = 375$ when $t = 4$ months and 8 months.

(ii)



$$\text{period} = \frac{2\pi}{\frac{\pi}{6}} = 12$$

♦ *Mean mark 39%*

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12. Trig Ratios, 2UA 2013 HSC 14c

Find $\angle ADB$

$$\angle ADB = \frac{\pi}{6} + \frac{\pi}{2} \quad \text{(exterior angle of } \Delta BDC\text{)}$$
$$= \frac{2\pi}{3} \text{ radians}$$

Find AD

$$\tan\left(\frac{\pi}{6}\right) = \frac{4}{BC}$$
$$\frac{1}{\sqrt{3}} = \frac{4}{BC}$$
$$BC = 4\sqrt{3}$$

Using Pythagoras:

$$AC^2 + BC^2 = AB^2$$
$$AC^2 + \left(4\sqrt{3}\right)^2 = 13^2$$
$$AC^2 = 169 - 48$$
$$= 121$$
$$\Rightarrow AC = 11$$
$$\therefore AD = AC - DC$$
$$= 11 - 4$$
$$= 7$$

Using sine rule:

$$\frac{AB}{\sin\angle BDA} = \frac{AD}{\sin x}$$
$$\frac{13}{\sin\left(\frac{2\pi}{3}\right)} = \frac{7}{\sin x}$$
$$13 \times \sin x = 7 \times \sin\left(\frac{2\pi}{3}\right)$$
$$\sin x = \frac{7}{13} \times \sin\left(\frac{2\pi}{3}\right)$$
$$= \frac{7}{13} \times \frac{\sqrt{3}}{2}$$
$$= \frac{7\sqrt{3}}{26}$$

$\therefore$ The exact value of $\sin x = \dfrac{7\sqrt{3}}{26}$.

♦ *Mean mark 36%.*
STRATEGY TIP: The hint to use the sine rule should flag to students that they will be dealing in non-right angled trig (i.e. ΔABD) and to direct their energies at initially finding $\angle ADB$ and AD.

13. Real Functions, 2UA 2011 HSC 6b

Find locus of $P(x, y)$

$P(x, y)$ $A(-1,0)$ $B(3, 0)$

$$\text{Dist } PA^2 = (y_2 - y_1)^2 + (x_2 - x_1)^2$$
$$= y^2 + (x+1)^2$$
$$\text{Dist } PB^2 = (y - 0)^2 + (x - 3)^2$$
$$= y^2 + (x-3)^2$$

Since $PA^2 + PB^2 = 40$

$$\Rightarrow y^2 + (x+1)^2 + y^2 + (x-3)^2 = 40$$
$$2y^2 + x^2 + 2x + 1 + x^2 - 6x + 9 = 40$$
$$2y^2 + 2x^2 - 4x + 10 = 40$$
$$y^2 + x^2 - 2x + 5 = 20$$
$$y^2 + (x-1)^2 + 4 = 20$$
$$y^2 + (x-1)^2 = 16$$

$\therefore$ $P(x, y)$ is a circle, centre $(1, 0)$, radius 4 units.

♦ *Mean mark 39%.*
MARKER'S COMMENT: Challenging question with many students unable to handle the algebra in expanding and completing the squares.

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