Thank you for subscribing to SmarterMaths Teacher Edition (Silver) in 2020.

The "2020 HSC Comprehensive Revision Series" provides around 12 hours of HSC revision questions for early starters. It has a weighting toward more difficult examples and is targeted at motivated students aiming for a Band 5 or 6 result. We recommend students **attempt, carefully review and annotate** this revision set in Term 3, and use it as the foundation of a concise and high quality revision resource.

As in previous years, our "Final Stretch HSC Revision Series", which is a shorter version of this revision set for late starters, will be available in early-September for the final weeks before the Standard 2 HSC exam.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create this revision set that ensures students cover a wide cross-section of the key areas.

IMPORTANT: If students have been exposed to many of the questions in these worksheets during the year, we say great. Many top performing students attest to the benefits of doing quality questions 2-3 times before the HSC. The resulting confidence and speed through the exam creates a virtuous cycle for peak performance.

HSC Final Study: ADVANCED MATHEMATICS

F1 Working With Functions (~9% historical contribution)

Key Areas addressed by this worksheet

Composite Functions

- A revision focus on finding the domain and range of composite functions;
- Questions have been selected taking direction from both the NESA sample Advanced exam and other States who have an exam history in this topic content.

- We regard this topic as highly examinable at a wide range of difficulty levels and that it should be a revision focus.

Further Functions and Relations

- Absolute value *equality* equations are well covered (noting that absolute value *inequality* equations have moved to the Ext1 course);
- Linear inequality equations remain fair game and an example of this low hanging fruit is included;
- Graphing absolute value equations is an area where we expect to be examined more in the new course and is well covered;
- Graph reflections are reviewed;
- Circle functions are reviewed;
- Proportional (non-linear) relationships are again touched on (these were also revised in *F1 Working With Functions (Part 1)*.

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Questions

1. **Functions, 2ADV F1 SM-Bank 5 MC**

   Let $g(x) = x^2 + 2x - 3$ and $f(x) = e^{2x+3}$.

   Then $f(g(x))$ is given by

   A. $e^{4x+6} + 2e^{2x+3} - 3$

   B. $2x^2 + 4x - 6$

   C. e^{2x^2+4x-3}

   D. e^{2x^2+4x-6}

2. **Functions, 2ADV F1 SM-Bank 13 MC**

   Which one of the following functions satisfies the functional equation $f(f(x)) = x$?

   A. $f(x) = 2 - x$

   B. $f(x) = x^2$

   C. $f(x) = 2\sqrt{x}$

   D. $f(x) = x - 2$

3. **Functions, 2ADV F1 SM-Bank 21 MC**

   A circle with centre $(a, -2)$ and radius 5 units has equation

   $x^2 - 6x + y^2 + 4y = b$ where a and b are real constants.

   The values of a and b are respectively

   A. −3 and 38

   B. 3 and 12

   C. −3 and −8

   D. 3 and 18

4. **Functions, 2ADV F1 SM-Bank 4 MC**

   The function $f(x)$ satisfies the functional equation $f(f(x)) = x$ for $\{x:$ all $x, \ x \neq 1\}$.

   The rule for the function is

   A. $f(x) = x + 1$

   B. $f(x) = x - 1$

   C. $f(x) = \dfrac{x - 1}{x + 1}$

   D. $f(x) = \dfrac{x + 1}{x - 1}$

5. **Functions, 2ADV F1 SM-Bank 6 MC**

   Let $f(x) = e^x + e^{-x}$.

   $f(2u)$ is equal to

   A. $f(u) + f(-u)$

   B. $2f(u)$

   C. $(f(u))^2 - 2$

   D. $(f(u))^2 + 2$

6. Functions, 2ADV F1 SM-Bank 14 MC

Let $g(x) = \log_2(x), \quad x > 0$

Which one of the following equations is true for all positive real values of x?

A. $2g(8x) = g(x^2) + 8$

B. $2g(8x) = g(x^2) + 6$

C. $2g(8x) = (g(x) + 8)^2$

D. $2g(8x) = g(2x) + 6$

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7. Functions, 2ADV F1 2011 HSC 1e

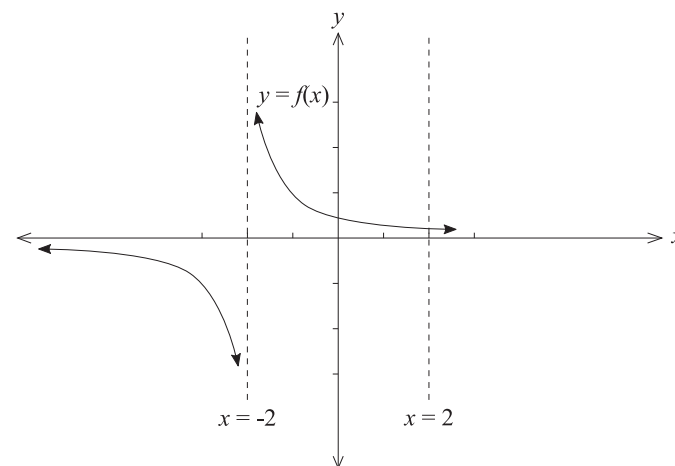Solve $2 - 3x \leq 8$. *(2 marks)*

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8. Functions, 2ADV F1 2008 HSC 1d

Solve $| 4x - 3 | = 7$. *(2 marks)*

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9. Functions, 2ADV F1 SM-Bank 37

Find all values of x for which $|x - 4| = \dfrac{x}{2} + 7$. *(3 marks)*

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10. Functions, 2ADV F1 SM-Bank 36

Consider the function $f(x) = \dfrac{1}{x + 2}$.

i. Sketch the graph $y = f(-x)$. *(2 marks)*

ii. On the same graph, sketch $y = -f(x)$. *(2 marks)*

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11. Functions, 2ADV F1 2019 HSC 13e

(i) Sketch the graph of $y = | x - 1 |$ for $-4 \leq x \leq 4$. *(1 mark)*
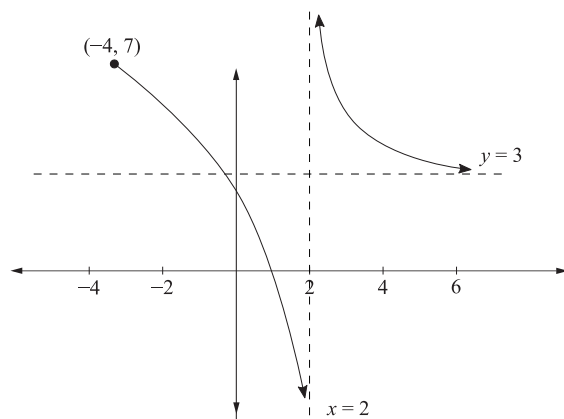
(ii) Using the sketch from part (i), or otherwise, solve $| x - 1 | = 2x + 4$. *(2 marks)*

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12. Functions, 2ADV F1 EQ-Bank 11

Given the function $f(x) = \sqrt{3 - x}$ and $g(x) = x^2 - 2$, sketch $y = g(f(x))$ over its natural domain. *(2 marks)*

13. Functions, 2ADV F1 EQ-Bank 6

The graph of $f(x)$ is shown below. It has asymptotes at $y = 3$ and $x = 2$.

$(-4, 7)$

$y = 3$

$-4 \quad -2 \quad 2 \quad 4 \quad 6$

$x = 2$

Using interval notation, state the domain and range of $f(x)$. *(2 marks)*

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14. Functions, 2ADV F1 EQ-Bank 7

The current of an electrical circuit, measured in amps (A), varies inversely with its resistance, measured in ohms (R).

When the resistance of a circuit is 28 ohms, the current is 3 amps.

What is the current when the resistance is 8 ohms? *(2 marks)*

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15. Functions, 2ADV F1 SM-Bank 31

Find the domain and range of $f(g(x))$ given

$f(x) = 2x^2 - 8x$ and $g(x) = x + 2$. *(2 marks)*

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16. Functions, 2ADV F1 2010 HSC 1c

Write down the equation of the circle with centre $(-1, 2)$ and radius 5. *(1 mark)*

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17. Functions, 2ADV F1 2017 HSC 11h

Find the domain of the function $f(x) = \sqrt{3 - x}$. *(2 marks)*

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18. Functions, 2ADV F1 SM-Bank 7

Let $f(x) = \log_e(x)$ for $x > 0$, and $g(x) = x^2 + 1$ for all x.

   i. Find $h(x)$, where $h(x) = f(g(x))$. *(1 mark)*

   ii. State the domain and range of $h(x)$. *(2 marks)*

   iii. Show that $h(x) + h(-x) = f\left((g(x))^2\right)$. *(2 marks)*

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19. Functions, 2ADV F1 SM-Bank 11

Given $f(x) = \sqrt{x^2 - 9}$ and $g(x) = x + 5$

a. Find integers c and d such that $f(g(x)) = \sqrt{(x + c)(x + d)}$ *(2 marks)*

b. State the domain for which $f(g(x))$ is defined. *(2 marks)*

Worked Solutions

1. Functions, 2ADV F1 SM-Bank 5 MC

By trial and error,

Consider: $f(x) = e^{2x^2+4x-3}$

$f(g(x)) = e^{2(x^2+2x-3)+3}$

$\qquad = e^{2x^2+4x-3}$

$\Rightarrow C$

2. Functions, 2ADV F1 SM-Bank 13 MC

By trial and error,

Consider: $f(x) = 2 - x$

$f(f(x)) = 2 - (2 - x)$

$\qquad = x$

$\Rightarrow A$

3. Functions, 2ADV F1 SM-Bank 21 MC

$x^2 - 6x + y^2 + 4y = b$

Completing the squares:

$x^2 - 6x + 3^2 - 9 + y^2 + 4y + 2^2 - 4 = b$

$(x - 3)^2 + (y + 2)^2 - 13 = b$

$(x - 3)^2 + (y + 2)^2 = b + 13$

$\therefore a = 3$

$\therefore b + 13 = 25 \quad \Rightarrow \quad b = 12$

$\Rightarrow B$

4. Functions, 2ADV F1 SM-Bank 4 MC

By trial and error:

Consider $f(x) = \dfrac{x + 1}{x - 1}$ ◆ *Mean mark 47%.*

$f(f(x)) = \dfrac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}$

$\qquad = \dfrac{x + 1 + x - 1}{x + 1 - x + 1}$

$\qquad = x$

$\Rightarrow D$

5. Functions, 2ADV F1 SM-Bank 6 MC

By trial and error,

Consider $(f(u))^2 - 2$: ◆ *Mean mark 44%.*

$f(2u) = e^{2u} + e^{-2u}$

$(f(u))^2 = \left(e^u + e^{-u}\right)^2$

$\qquad = e^{2u} + 2 + e^{-2u}$

$\therefore f(2u) = (f(u))^2 - 2$

$\Rightarrow C$

6. Functions, 2ADV F1 SM-Bank 14 MC

Consider Option B:

$\text{LHS} = 2g(8x)$

$\qquad = 2\log_2(8x)$

$\qquad = 2\log_2(8) + 2\log_2(x)$

$\qquad = 2\log_2(2^3) + 2\log_2(x)$

$\qquad = 6 + \log_2(x^2)$

$\qquad = g(x^2) + 6$

$\Rightarrow \quad B$

---

7. Functions, 2ADV F1 2011 HSC 1e

$2 - 3x \le 8$

$\quad -3x \le 6$

$\qquad x \ge -\dfrac{6}{3}$

$\qquad x \ge -2$

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8. Functions, 2ADV F1 2008 HSC 1d

$|\,4x - 3\,| = 7$

$4x - 3 = 7 \qquad -(4x - 3) = 7$

$\quad 4x = 10 \qquad\ -4x + 3 = 7$

$\qquad x = \dfrac{5}{2} \qquad\qquad -4x = 4$

$\qquad\qquad\qquad\qquad\quad x = -1$

$\therefore x = \dfrac{5}{2} \ \text{ or } \ -1$

---

9. Functions, 2ADV F1 SM-Bank 37

$x - 4 = \dfrac{x}{2} + 7 \qquad \text{or} \qquad -(x - 4) = \dfrac{x}{2} + 7$

$2x - 8 = x + 14 \qquad\qquad -2x + 8 = x + 14$

$\qquad x = 22 \qquad\qquad\qquad\qquad 3x = -6$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad x = -2$

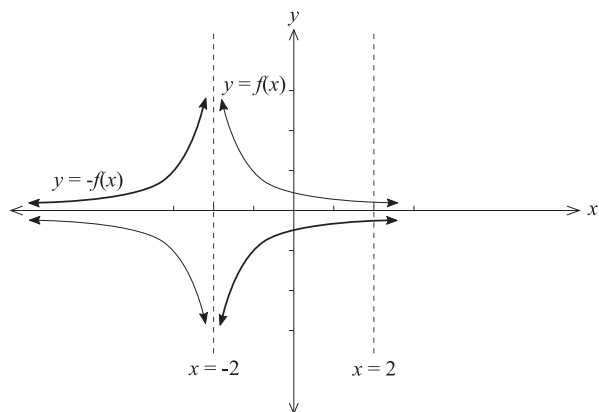$\therefore x = 22 \ \text{ or } -2$

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10. Functions, 2ADV F1 SM-Bank 36
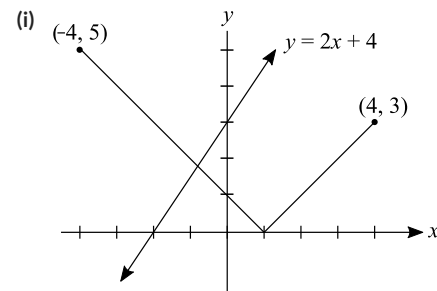
i. Sketch $y = \dfrac{1}{x+2}$

$y = f(-x) \;\Rightarrow\;$ reflect $y = \dfrac{1}{x+2}$ in the y-axis.
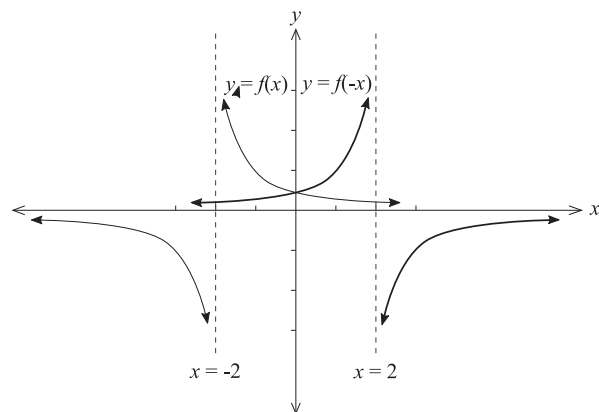
ii. $y = -f(x) \;\Rightarrow\;$ reflect $y = \dfrac{1}{x+2}$ in the x-axis.

11. Functions, 2ADV F1 2019 HSC 13e

(i)

ii. By inspection, intersection when $x = -1$

Test:

$|-1-1| = -2 + 4$

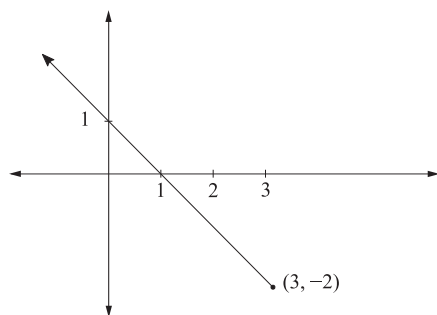$2 = 2$

$\therefore$ Intersection at $(-1, 2)$

12. Functions, 2ADV F1 EQ-Bank 11

$$g(x) = x^2 - 2, \quad f(x) = \sqrt{3 - x}$$

$$g(f(x)) = \left(\sqrt{3 - x}\right)^2 - 2$$

$$= 3 - x - 2$$

$$= 1 - x$$

Since $f(x) = \sqrt{3 - x}$,

$\Rightarrow$ Domain: $x \leq 3$

$(3, -2)$

13. Functions, 2ADV F1 EQ-Bank 6

Domain: $[-4, 2) \cup (2, \infty)$

Range: $(-\infty, \infty)$

14. Functions, 2ADV F1 EQ-Bank 7

$$A \propto \frac{1}{R}$$

$$A = \frac{k}{R}$$

When $A = 3, \ R = 28$

$$3 = \frac{k}{28}$$

$$k = 84$$

Find $A \ when \ R = 8$:

$$A = \frac{84}{8}$$

$$= 10.5$$

15. Functions, 2ADV F1 SM-Bank 31

$$f(g(x)) = 2(x + 2)^2 - 8(x + 2)$$

$$= 2\left(x^2 + 4x + 4\right) - 8x - 16$$

$$= 2x^2 + 8x + 8 - 8x - 16$$

$$= 2\left(x^2 - 4\right)$$

$\therefore$ Domain: all x

$\therefore$ Range: $-8 \leq y < \infty$

16. Functions, 2ADV F1 2010 HSC 1c

Circle with centre $(-1, 2), \ r = 5$

$$(x + 1)^2 + (y - 2)^2 = 25$$

MARKER'S COMMENT:
Expanding this equation is not necessary!

17. Functions, 2ADV F1 2017 HSC 11h

Solution 1

Domain of $f(x) = \sqrt{3 - x}$

$3 - x \geq 0$

$x \leq 3$

Note domain can also be expressed as: $(-\infty, 3]$

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18. Functions, 2ADV F1 SM-Bank 7

i. $h(x) = f\left(x^2 + 1\right)$

$= \log_e\left(x^2 + 1\right)$

ii. Domain $(h) = $ Domain (g): all x

$\Rightarrow x^2 + 1 \geq 1$

$\Rightarrow \log_e\left(x^2 + 1\right) \geq 0$

$\therefore$ Range $h(x)$: $h \geq 0$

♦♦ Mean mark part (a)(ii) 30%.

iii. LHS $= h(x) + h(-x)$

$= \log_e\left(x^2 - 1\right) + \log_e\left((-x)^2 + 1\right)$

$= \log_e\left(x^2 + 1\right) + \log_e\left(x^2 + 1\right)$

$= 2\log_e\left(x^2 + 1\right)$

MARKER'S COMMENT: Many students were unsure of how to present their working in this question. Note the layout in the solution.
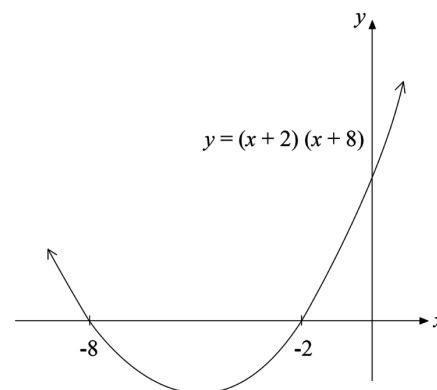
RHS $= f\left(\left(x^2 + 1\right)^2\right)$

$= 2\log_e\left(x^2 + 1\right)$

$\therefore h(x) + h(-x) = f\left((g(x))^2\right)$ ... as required

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19. Functions, 2ADV F1 SM-Bank 11

a. $f(g(x)) = \sqrt{(x + 5)^2 - 9}$

$= \sqrt{x^2 + 10x + 16}$

$= \sqrt{(x + 2)(x + 8)}$

$\therefore c = 2, d = 8$ or $c = 8, d = 2$

b. Find x such that:

$(x + 2)(x + 8) \geq 0$

♦♦♦ Mean mark 13%.
MARKER'S COMMENT: "Very poorly answered" with a common response of $-3 \leq x \leq 3$ that ignored the information from part (a).

$y = (x + 2)(x + 8)$

$(x + 2)(x + 8) \geq 0$ when

$x \leq -8$ or $x \geq -2$

$\therefore$ Domain: $x \leq -8$ and $x \geq -2$