√SmarterMaths

Thank you for subscribing to SmarterMaths Teacher Edition (Silver) in 2020.

The “2020 HSC Comprehensive Revision Series” provides an extensive set of cherry picked HSC revision questions for students starting their revision early. It has a weighting toward more difficult examples and is targeted at motivated students aiming for a Band 5 or 6 result. We recommend students ***attempt, carefully review and annotate*** this revision set in Term 3, and use it as the foundation of a concise and high quality revision resource.

As in previous years, our “Final Stretch HSC Revision Series”, which is a shorter version of this revision set for late starters, will be available in early-September for the final weeks before the Standard 2 HSC exam.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create this revision set that ensures students cover a wide cross-section of the key areas.

IMPORTANT: If students have been exposed to many of the questions in these worksheets during the year, we say great. Many top performing students attest to the benefits of doing quality questions 2-3 times before the HSC. The resulting confidence and speed through the exam creates a virtuous cycle for peak performance.

HSC Final Study: EXTENSION 1 MATHEMATICS

TRIGONOMETRY

Key Areas addressed by this worksheet:

T1 Inverse Trig Functions (Y11)

- Examined in 8 of the last 10 Ext1 papers, contributing an average of 2.1% per exam. Multiple choice has been a popular question style for this topic and numerous past MC questions are reviewed;
- Inverse trig graphs (the most common question type) are covered;
- Students are exposed to the notation of $\arccos x$ etc…

T2 Compound and Double Angles (Y11)

- A small area for dedicated questions although it requires skills that are used in numerous other syllabus topics;
- Drawing right angled triangles to solve equations is a feature of worked solutions. A selection of harder examples are reviewed, including *2013 HSC Q8 MC* which caused problems for many;
- NESA exemplar questions required students to construct compound angles from exact trig ratios - a number of questions cover this question type.

T3 Trig Equations (Y12)

- Auxiliary angles have been examined with a dedicated question 5 times in the last 7 years with allocations of 1-3 marks;
- Students must have clear and precise answer structure in their mind when tackling these questions and the worked solutions provide a reference for this;
- Examples with varying difficulty levels are selected in this revision, including the more challenging *2010 HSC Q4b* which caused problems;
- Proving trig identities is a very broad area and numerous questions of varying difficulty have been selected for review;
- \`t\` formula covered;
- We review *T3 EQ-Bank 3* where “all” solutions to a trig equation are required (note the general solution is not required for expressing the answer).

EXTENSION 1 MATHS:
2020 HSC Comprehensive Revision Series
- TRIGONOMETRY

T1 Inverse Trig Functions (Y11)

T2 Compound and Double Angles (Y11)

T3 Trig Equations

Teacher: Smarter Maths
Exam Equivalent Time: 90 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

Questions

1. Trigonometry, EXT1 T2 2016 HSC 3 MC

Which expression is equivalent to $\dfrac{\tan 2x - \tan x}{1 + \tan 2x \tan x}$?

(A) $\tan x$

(B) $\tan 3x$

(C) $\dfrac{\tan 2x - 1}{1 + \tan 2x}$

(D) $\dfrac{\tan x}{1 + \tan 2x \tan x}$

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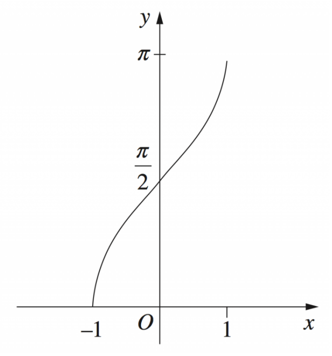
2. Trigonometry, EXT1 T1 2013 HSC 9 MC

The diagram shows the graph of a function.



Which function does the graph represent?

(A) $y = \cos^{-1} x$

(B) $y = \dfrac{\pi}{2} + \sin^{-1} x$

(C) $y = -\cos^{-1} x$

(D) $y = -\dfrac{\pi}{2} - \sin^{-1} x$

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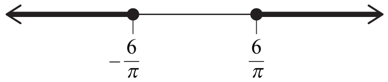
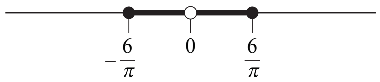
3. Trigonometry, EXT1 T1 2017 HSC 7 MC

Which diagram represents the domain of the function $f(x) = \sin^{-1}\left(\frac{3}{x}\right)$?

A. 

B. 

C. 

D. 

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4. Trigonometry, EXT1 T3 2017 HSC 4 MC

What is the value of $\tan\alpha$ when the expression $2\sin x - \cos x$ is written in the form $\sqrt{5}\sin(x - \alpha)$?

A. -2

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 2

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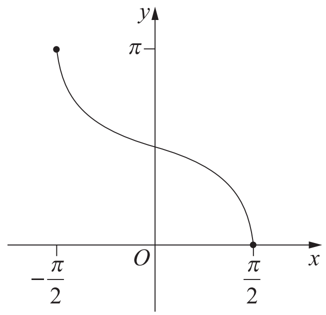
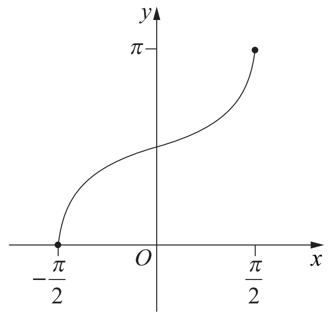
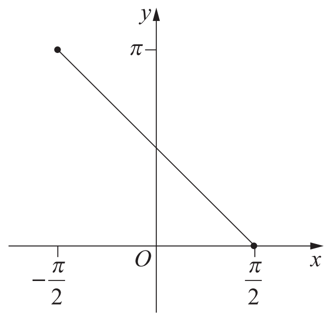
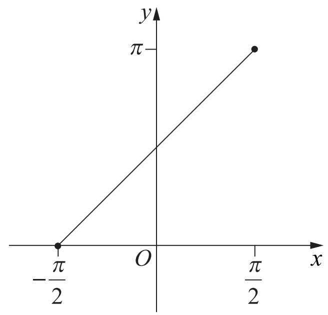
5. Trigonometry, EXT1 T1 2019 HSC 9 MC

Which graph best represents $y = \cos^{-1}(-\sin x)$, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$?

A. 

B. 

C. 

D. 

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6. Trigonometry, EXT1 T2 2019 HSC 6 MC

It is given that $\sin x = \frac{1}{4}$, where $\frac{\pi}{2} < x < \pi$.

What is the value of $\sin 2x$?

A. $-\frac{7}{8}$

B. $-\frac{\sqrt{15}}{8}$

C. $\frac{\sqrt{15}}{8}$

D. $\frac{7}{8}$

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7. Trigonometry, EXT1 T2 2013 HSC 8 MC

The angle θ satisfies $\sin\theta = \dfrac{5}{13}$ and $\dfrac{\pi}{2} < \theta < \pi$.

What is the value of $\sin 2\theta$?

(A) $\dfrac{10}{13}$

(B) $-\dfrac{10}{13}$

(C) $\dfrac{120}{169}$

(D) $-\dfrac{120}{169}$

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8. Trigonometry, EXT1 T1 2014 HSC 11c

Sketch the graph $y = 6\tan^{-1} x$, clearly indicating the range. *(2 marks)*

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9. Trigonometry, EXT1 T2 2006 HSC 1d

i. Simplify $(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)$ *(1 mark)*

ii. Hence, or otherwise, express $\dfrac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} - 1$, in its simplest form for $0 < \theta < \dfrac{\pi}{2}$. *(2 marks)*

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10. Trigonometry, EXT1 T1 EQ-Bank 1

Show that $\cos(\sin^{-1} q) = \sqrt{1 - q^2}$ *(2 marks)*

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11. Trigonometry, EXT1 T1 2007 HSC 5c

Find the exact values of x and y which satisfy the simultaneous equations

$\sin^{-1} x + \dfrac{1}{2}\cos^{-1} y = \dfrac{\pi}{3}$ and

$3\sin^{-1} x - \dfrac{1}{2}\cos^{-1} y = \dfrac{2\pi}{3}$. *(3 marks)*

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12. Trigonometry, EXT1 T1 2011 HSC 2d

Sketch the graph of the function $f(x) = 2\arccos x$. Clearly indicate the domain and range of the function. *(2 marks)*

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13. Trigonometry, EXT1 T1 2012 HSC 13a

Write $\sin\left(2\cos^{-1}\left(\dfrac{2}{3}\right)\right)$ in the form $a\sqrt{b}$, where a and b are rational. *(2 mark)*

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14. Trigonometry, EXT1 T3 2007 HSC 2a

By using the substitution $t = \tan\dfrac{\theta}{2}$, or otherwise, show that $\dfrac{1 - \cos\theta}{\sin\theta} = \tan\dfrac{\theta}{2}$. *(2 marks)*

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15. Trigonometry, EXT1 T3 2009 HSC 2b

i. Express $3\sin x + 4\cos x$ in the form $A\sin(x + \alpha)$ where $0 \leq \alpha \leq \dfrac{\pi}{2}$. *(2 marks)*

ii. Hence, or otherwise, solve $3\sin x + 4\cos x = 5$ for $0 \leq x \leq 2\pi$.
Give your answer, or answers, correct to two decimal places. *(2 marks)*

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16. Trigonometry, EXT1 T2 SM-Bank 2

Find the exact value of $\cos\dfrac{\pi}{8}$. *(2 marks)*

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17. Trigonometry, EXT1 T2 EQ-Bank 5

If $\sin\theta = -\dfrac{4}{6}$ and $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$,

determine the exact value of $\cos\theta$ in its simplest form. *(2 marks)*

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18. Trigonometry, EXT1 T3 SM-Bank 12

Solve $\sin(2x) = \sin x$, for $0 \le x \le 2\pi$. *(3 marks)*

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19. Trigonometry, EXT1 T3 EQ-Bank 3

Find all values of θ that satisfy the equation $\sqrt{3}\cos\theta = \sin(2\theta)$. *(3 marks)*

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20. Trigonometry, EXT1 T3 EQ-Bank 4

The current flowing through an electrical circuit can be modelled by the function

$$f(t) = 6\sin 0.05t + 8\cos 0.05t, \quad t \ge 0$$

i. Express the function in the form $f(t) = A\sin(at + b)$, for $0 \le b \le \dfrac{\pi}{2}$. *(2 marks)*

ii. Find the time at which the current first obtains it maximum value. *(1 mark)*

iii. Sketch the graph of $f(t)$. Clearly show its range and label the coordinates of its first maximum value. Do not label x-intercepts. *(1 mark)*

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21. Trigonometry, EXT1 T3 2010 HSC 4b

i. Express $2\cos\theta + 2\cos\left(\theta + \dfrac{\pi}{3}\right)$ in the form $R\cos(\theta + \alpha)$,

where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. *(3 marks)*

ii. Hence, or otherwise, solve $2\cos\theta + 2\cos\left(\theta + \dfrac{\pi}{3}\right) = 3$,

for $0 < \theta < 2\pi$. *(2 marks)*

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22. Trigonometry, EXT1 T3 EQ-Bank 1

i. Show that $\sin x + \sin 3x = 2\sin 2x\cos x$. *(2 marks)*

ii. Hence or otherwise, find all values of x that satisfy

$$\sin x + \sin 2x + \sin 3x = 0, \quad x \in [0, 2\pi].$$ *(2 marks)*

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23. Trigonometry, EXT1 T3 2005 HSC 4b

By making the substitution $t = \tan\dfrac{\theta}{2}$, or otherwise, show that

$$\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}.$$ *(2 marks)*

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24. Trigonometry, EXT1 T3 2008 HSC 6b

It can be shown that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ for all values of θ. (Do NOT prove this.)

Use this result to solve $\sin 3\theta + \sin 2\theta = \sin\theta$ for $0 \le \theta \le 2\pi$. *(3 marks)*

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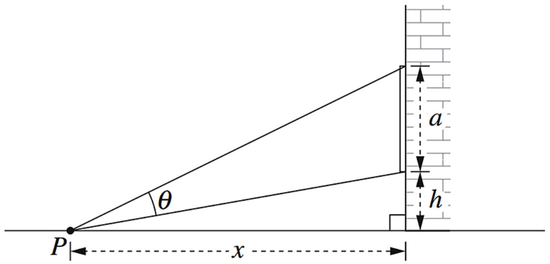
25. Trigonometry, EXT1 T3 SM-Bank 1

A billboard of height a metres is mounted on the side of a building, with its bottom edge h metres above street level. The billboard subtends an angle θ at the point P, x metres from the building.



Use the identity $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$ to show that

$$\theta = \tan^{-1}\left[\frac{ax}{x^2 + h(a+h)}\right].$$ *(2 marks)*

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26. Trigonometry, EXT1 T1 SM-Bank 3

Determine the exact value of

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \cos^{-1}\left(-\frac{1}{2}\right).$$ *(3 marks)*

Worked Solutions

1. Trigonometry, EXT1 T2 2016 HSC 3 MC

$$\frac{\tan 2x - \tan x}{1 + \tan 2x \tan x}$$
$$= \tan(2x - x)$$
$$= \tan x$$
$$\Rightarrow \ A$$

2. Trigonometry, EXT1 T1 2013 HSC 9 MC

By elimination,

The graph passes through $(1, \pi)$

The only equation to satisfy this point is

$$y = \frac{\pi}{2} + \sin^{-1} x$$
$$\Rightarrow \ B$$

3. Trigonometry, EXT1 T1 2017 HSC 7 MC

$$f(x) = \sin^{-1}\left(\frac{3}{x}\right)$$

$$\frac{3}{x} \geq -1 \quad \text{and} \quad \frac{3}{x} \leq 1$$
$$\frac{x}{3} \leq -1 \qquad\qquad x \geq 3$$
$$x \leq -3$$
$$\Rightarrow A$$

4. Trigonometry, EXT1 T3 2017 HSC 4 MC

Using $\sqrt{5}\sin(x-\alpha) = \sqrt{5}\sin x\cos\alpha - \sqrt{5}\cos x\sin\alpha$,

$\Rightarrow \sqrt{5}\sin x\cos\alpha - \sqrt{5}\cos x\sin\alpha = 2\sin x - \cos x$

$\Rightarrow \sqrt{5}\cos\alpha = 2,\ \sqrt{5}\sin\alpha = 1$

$\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{1}{2}$

$\therefore \tan\alpha = \dfrac{1}{2}$

$\Rightarrow C$

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5. Trigonometry, EXT1 T1 2019 HSC 9 MC

$$\begin{aligned} y &= \cos^{-1}(-\sin x) \\ &= \cos^{-1}\left(\cos\left(x+\frac{\pi}{2}\right)\right) \\ &= x+\frac{\pi}{2} \end{aligned}$$

$\Rightarrow D$

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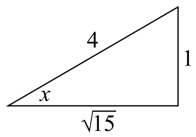
6. Trigonometry, EXT1 T2 2019 HSC 6 MC

$\sin x = \dfrac{1}{4}$



$\cos x = -\dfrac{\sqrt{15}}{4},\ \left(\dfrac{\pi}{2} < x < \pi\right)$

$$\begin{aligned} \sin 2x &= 2\sin x\cos x \\ &= 2\times\frac{1}{4}\times -\frac{\sqrt{15}}{4} \\ &= -\frac{\sqrt{15}}{8} \end{aligned}$$

$\Rightarrow B$

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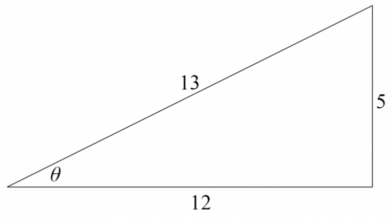
7. Trigonometry, EXT1 T2 2013 HSC 8 MC

♦ *Mean mark 44%*

$$\sin\theta = \frac{5}{13} \quad \left(\frac{\pi}{2} < \theta < \pi\right)$$



Since θ is in the 2nd quadrant,

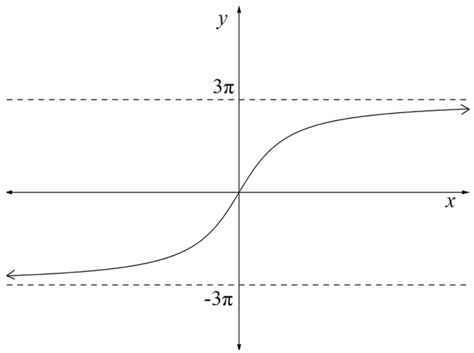
$$\cos\theta = -\frac{12}{13}$$

$$\begin{aligned}\therefore \sin 2\theta &= 2\sin\theta\cos\theta \\ &= 2 \times \frac{5}{13} \times -\frac{12}{13} \\ &= -\frac{120}{169}\end{aligned}$$

$\Rightarrow D$

8. Trigonometry, EXT1 T1 2014 HSC 11c

$$y = 6\tan^{-1} x$$



9. Trigonometry, EXT1 T2 2006 HSC 1d

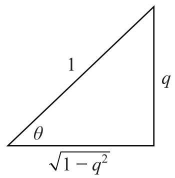
i. $(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)$

$$\begin{aligned}&= \sin^3\theta - \sin^2\theta\cos\theta + \sin\theta\cos^2\theta + \cos\theta\sin^2\theta - \sin\theta\cos^2\theta + \cos^3\theta \\ &= \sin^3\theta + \cos^3\theta\end{aligned}$$

ii. $\dfrac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} - 1$

$$\begin{aligned}&= \frac{(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}{\sin\theta + \cos\theta} - 1 \\ &= \sin^2\theta + \cos^2\theta - \sin\theta\cos\theta - 1 \\ &= 1 - \sin\theta\cos\theta - 1 \\ &= -\sin\theta\cos\theta\end{aligned}$$

10. Trigonometry, EXT1 T1 EQ-Bank 1



$$\begin{aligned}\sin\theta &= q \\ \theta &= \sin^{-1} q\end{aligned}$$

$$\begin{aligned}\therefore \cos\left(\sin^{-1} q\right) &= \cos\theta \\ &= \sqrt{1 - q^2}\end{aligned}$$

11. Trigonometry, EXT1 T1 2007 HSC 5c

$$\sin^{-1} x + \frac{1}{2}\cos^{-1} y = \frac{\pi}{3} \quad \ldots (1)$$
$$3\sin^{-1} x - \frac{1}{2}\cos^{-1} y = \frac{2\pi}{3} \quad \ldots (2)$$

Add $(1) + (2)$

$$4\sin^{-1} x = \pi$$
$$\sin^{-1} x = \frac{\pi}{4}$$
$$\therefore x = \frac{1}{\sqrt{2}}$$

MARKER'S COMMENT: The use of ther elimination method here proved much more successful than substitution.

Substitute $x = \dfrac{1}{\sqrt{2}}$ into (1)

$$\sin^{-1}\frac{1}{\sqrt{2}} + \frac{1}{2}\cos^{-1} y = \frac{\pi}{3}$$
$$\frac{\pi}{4} + \frac{1}{2}\cos^{-1} y = \frac{\pi}{3}$$
$$\frac{1}{2}\cos^{-1} y = \frac{\pi}{12}$$
$$\cos^{-1} y = \frac{\pi}{6}$$
$$\therefore y = \frac{\sqrt{3}}{2}$$

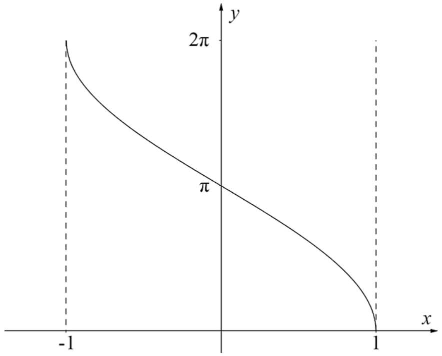
12. Trigonometry, EXT1 T1 2011 HSC 2d

$$y = 2\cos^{-1} x$$
$$\frac{y}{2} = \cos^{-1} x$$

Domain $-1 \le x \le 1$

Since $0 \le \dfrac{y}{2} \le \pi$

Range $0 \le y \le 2\pi$

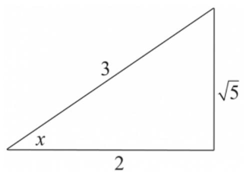


13. Trigonometry, EXT1 T1 2012 HSC 13a

$$\sin\left(2\cos^{-1}\left(\frac{2}{3}\right)\right)$$

Let $x = \cos^{-1}\left(\frac{2}{3}\right)$

$\therefore\ \cos x = \frac{2}{3}$

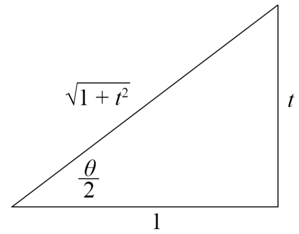


$\sin x = \frac{\sqrt{5}}{3}$

$$\begin{aligned}\sin\left(2\cos^{-1}\left(\frac{2}{3}\right)\right) &= \sin 2x \\ &= 2\sin x\cos x \\ &= 2 \times \frac{\sqrt{5}}{3} \times \frac{2}{3} \\ &= \frac{4}{9}\sqrt{5}\end{aligned}$$

TIP: This question is made less complicated by reminding yourself that $\cos^{-1}\left(\frac{2}{3}\right)$ is simply an angle.

14. Trigonometry, EXT1 T3 2007 HSC 2a



$\Rightarrow \tan\frac{\theta}{2} = t$

$\Rightarrow \sin\theta = 2 \cdot \frac{t}{\sqrt{1+t^2}} \cdot \frac{1}{\sqrt{1+t^2}} = \frac{2t}{1+t^2}$

$\Rightarrow \cos\theta = \frac{1}{1+t^2} - \frac{t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$

Show $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$:

$$\begin{aligned}\frac{1-\cos\theta}{\sin\theta} &= \frac{1-\left(\frac{1-t^2}{1+t^2}\right)}{\frac{2t}{1+t^2}} \times \frac{1+t^2}{1+t^2} \\ &= \frac{1+t^2-\left(1-t^2\right)}{2t} \\ &= \frac{2t^2}{2t} \\ &= t \\ &= \tan\frac{\theta}{2}\ \ldots\text{as required}\end{aligned}$$

COMMENT: The values of $\sin\theta$ and $\cos\theta$ can be committed to memory or quickly derived using double angle formulae (as shown in the worked solution).

15. Trigonometry, EXT1 T3 2009 HSC 2b

i. $A\sin(x+\alpha) = 3\sin x + 4\cos x$

$A\sin x\cos\alpha + A\cos x\sin\alpha = 3\sin x + 4\cos x$

$$\Rightarrow A\cos\alpha = 3 \quad A\sin\alpha = 4$$

$$A^2 = 3^2 + 4^2 = 25$$

$$\therefore A = 5$$

$$\Rightarrow 5\cos\alpha = 3$$

$$\cos\alpha = \frac{3}{5}$$

$$\alpha = \cos^{-1}\left(\frac{3}{5}\right) = 0.9272\ldots \text{ radians}$$

$$\therefore\ 3\sin x + 4\cos x = 5\sin\left(x + \cos^{-1}\left(\frac{3}{5}\right)\right)$$

ii. $3\sin x + 4\cos x = 5$

$$5\sin(x+\alpha) = 5$$

$$\sin(x+\alpha) = 1$$

$$x + \alpha = \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$x = \frac{\pi}{2} - 0.9272\ldots \quad (0 \le x \le 2\pi)$$

$$= 0.6435\ldots$$

$$= 0.64 \text{ radians (2 d.p.)}$$

16. Trigonometry, EXT1 T2 SM-Bank 2

Using: $\cos 2A = 2\cos^2 A - 1$

$$2\cos^2\frac{\pi}{8} - 1 = \cos\frac{\pi}{4}$$

$$2\cos^2\frac{\pi}{8} = \frac{1}{\sqrt{2}} + 1$$

$$\cos^2\frac{\pi}{8} = \frac{1+\sqrt{2}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

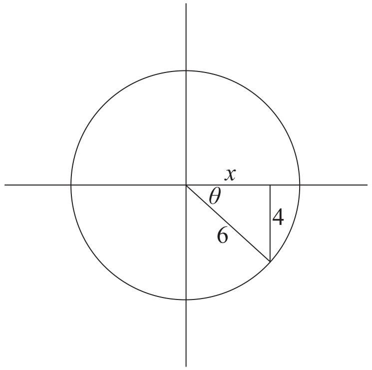
$$= \frac{\sqrt{2}+2}{4}$$

$$\therefore \cos\frac{\pi}{8} = \sqrt{\frac{\sqrt{2}+2}{4}}$$

$$= \frac{\sqrt{\sqrt{2}+2}}{2}$$

17. Trigonometry, EXT1 T2 EQ-Bank 5

Consider the angle graphically:



COMMENT: Pay careful attention to the range of θ.

Since $\sin\theta$ is negative $\Rightarrow \underbrace{\text{4th quadrant}}_{-\frac{\pi}{2}<\theta<\frac{\pi}{2}}$

Using Pythagoras,

$x^2 = 6^2 - 4^2$

$x = \sqrt{20} = 2\sqrt{5}$

$$\therefore \cos\theta = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$$

18. Trigonometry, EXT1 T3 SM-Bank 12

$2\sin x\cos x = \sin x$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0,\ \cos x = \frac{1}{2}$

If $\sin x = 0 \Rightarrow x = 0,\ \pi,\ 2\pi$

If $\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

MARKER'S COMMENT: Many students expanded $\sin(2x)$ and then cancelled $\sin x$ on both sides which lost a set of solutions!

19. Trigonometry, EXT1 T3 EQ-Bank 3

$$\sqrt{3}\cos\theta = \sin(2\theta)$$

$$2\cos\theta\sin\theta - \sqrt{3}\cos\theta = 0$$

$$\cos\theta\left(\sin\theta - \sqrt{3}\right) = 0$$

$\cos\theta = 0$ or $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 90°, 270°$ or $\theta = 60°, 120°$ $(0° \le \theta \le 360°)$

COMMENT: Expressing "all values" is specifically mentioned in Topic Guidance as an application of arithmetic sequences.

$\therefore \theta = 90° + m \times 180°,\ 60° + m \times 360°$ or $120° + m \times 360°$

where m is an arbitrary integer.

20. Trigonometry, EXT1 T3 EQ-Bank 4

i. $f(t) = 6\sin 0.05t + 8\cos 0.05t$

$$\begin{aligned} A\sin(at+b) &= A\sin(0.05t+b) \\ &= A\sin(0.05t)\cos(b) + A\cos(0.05t)\sin(b) \end{aligned}$$

$$\Rightarrow A\cos(b) = 6,\ A\sin(b) = 8$$

$$A^2 = 6^2 + 8^2$$

$$A = 10$$

$$\Rightarrow 10\cos(b) = 6$$

$$\cos(b) = \frac{6}{10}$$

$$b = \cos^{-1} 0.06 \approx 0.927 \text{ radians}$$

$$\therefore f(t) = 10\sin(0.05t + 0.927)$$

ii. Max occurs at $\sin(0.05t + 0.927) = \sin\dfrac{\pi}{2}$

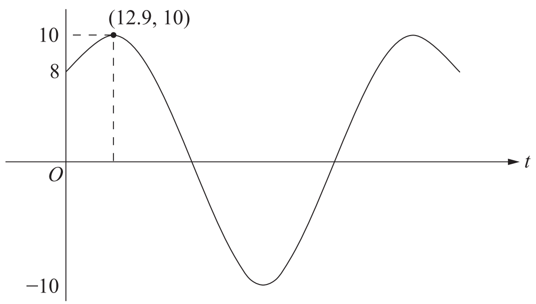
$$0.05t + 0.927 = \frac{\pi}{2}$$

$$0.05t = 0.643\ldots$$

$$\therefore t = 12.87$$

$$= 12.9 \text{ (to 1 d.p.)}$$

iii.



21. Trigonometry, EXT1 T3 2010 HSC 4b

i. $2\cos\theta + 2\cos\left(\theta + \dfrac{\pi}{3}\right)$

$$\begin{aligned} &= 2\cos\theta + 2\left(\cos\theta\cos\left(\frac{\pi}{3}\right) - \sin\theta\sin\left(\frac{\pi}{3}\right)\right) \\ &= 2\cos\theta + 2\cos\theta \times \frac{1}{2} - 2\sin\theta \times \frac{\sqrt{3}}{2} \\ &= 2\cos\theta + \cos\theta - \sqrt{3}\sin\theta \\ &= 3\cos\theta - \sqrt{3}\sin\theta \end{aligned}$$

$$R\cos(\theta+\alpha) = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

$$R\cos\alpha = 3 \qquad R\sin\alpha = \sqrt{3}$$

$$\cos\alpha = \frac{3}{R} \qquad \sin\alpha = \frac{\sqrt{3}}{R}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\therefore \alpha = \frac{\pi}{6} \quad \left(0 < \alpha < \frac{\pi}{2}\right)$$

$$\begin{aligned} R^2 &= 3^2 + \left(\sqrt{3}\right)^2 \\ &= 9 + 3 \\ R &= \sqrt{12} = 2\sqrt{3} \end{aligned}$$

$$\therefore\ 2\cos\theta + 2\cos\left(\theta + \frac{\pi}{3}\right) = 2\sqrt{3}\cos\left(\theta + \frac{\pi}{6}\right)$$

ii. $2\cos\theta + 2\cos\left(\theta + \dfrac{\pi}{3}\right) = 3$

$$2\sqrt{3}\cos\left(\theta + \frac{\pi}{6}\right) = 3$$

$$\cos\left(\theta + \frac{\pi}{6}\right) = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

♦ *Mean mark part (ii) 49%*
MARKER'S COMMENT: Many students did not check their answers against the stated domain for θ.

Since cos is positive in 1st and 4th quadrants,

$$\theta + \frac{\pi}{6} = \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$$

$$\therefore \theta = \frac{5\pi}{3} \quad (0 < \theta < 2\pi)$$

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22. Trigonometry, EXT1 T3 EQ-Bank 1

i.
$$\begin{aligned}\sin x + \sin 3x &= \sin x + \sin 2x\cos x + \cos 2x \sin x\\ &= \sin x + 2\sin x\cos^2 x + \left(\cos^2 x - \sin^2 x\right)\sin x\\ &= \sin x + 2\sin x\cos^2 x + \cos^2 x\sin x - \sin^3 x\\ &= \sin x + 3\sin x\left(1 - \sin^2 x\right) - \sin^3 x\\ &= \sin x + 3\sin x - 3\sin^3 x - \sin^3 x\\ &= 4\sin x\left(1 - \sin^2 x\right)\\ &= 4\sin x\cos^2 x\\ &= 2\sin 2x\cos x\\ &= \text{RHS}\end{aligned}$$

ii.
$$\sin x + \sin 2x + \sin 3x = 0$$
$$\sin 2x + 2\sin 2x\cos x = 0$$
$$\sin 2x(1 + 2\cos x) = 0$$

If $\sin 2x = 0$:

$$2x = 0, \pi, 2\pi, 3\pi, 4\pi$$
$$\therefore x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

If $\cos x = -\frac{1}{2}$:

$$\therefore x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

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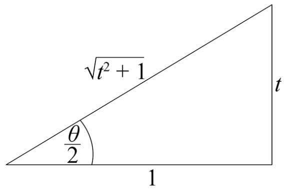
23. Trigonometry, EXT1 T3 2005 HSC 4b

Show $\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}$



$$\tan\frac{\theta}{2} = t$$

$$\Rightarrow \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{2t}{t^2+1}$$

$$\Rightarrow \cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = \frac{1-t^2}{t^2+1}$$

$$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2t}{1-t^2}$$

$$\begin{aligned}\text{LHS} &= \frac{1}{\sin\theta} + \frac{1}{\tan\theta}\\ &= \frac{t^2+1}{2t} + \frac{1-t^2}{2t}\\ &= \frac{t^2+1+1-t^2}{2t}\\ &= \frac{1}{t}\\ &= \frac{1}{\tan\frac{\theta}{2}}\\ &= \cot\frac{\theta}{2}\\ &= \text{RHS} \ \ldots \text{ as required.}\end{aligned}$$

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24. Trigonometry, EXT1 T3 2008 HSC 6b

Substitute $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

into $\sin 3\theta + \sin 2\theta = \sin\theta$,

$$\begin{aligned}
3\sin\theta - 4\sin^3\theta + \sin 2\theta &= \sin\theta \\
2\sin\theta - 4\sin^3\theta + 2\sin\theta\cos\theta &= 0 \\
2\sin\theta\left[1 - 2\sin^2\theta + \cos\theta\right] &= 0 \\
2\sin\theta\left[1 - 2\left(1 - \cos^2\theta\right) + \cos\theta\right] &= 0 \\
2\sin\theta\left[1 - 2 + 2\cos^2\theta + \cos\theta\right] &= 0 \\
2\sin\theta\left[2\cos^2\theta + \cos\theta - 1\right] &= 0 \\
2\sin\theta(2\cos\theta - 1)(\cos\theta + 1) &= 0
\end{aligned}$$

$2\sin\theta = 0$

$\Rightarrow \theta = 0, \pi, 2\pi$

$2\cos\theta - 1 = 0$

$\cos\theta = \dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

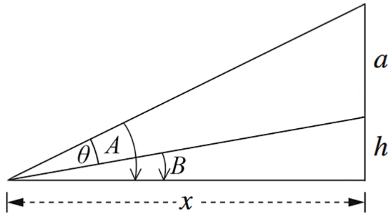
$\cos\theta + 1 = 0$

$\cos\theta = -1$

$\Rightarrow \theta = \pi$

$$\therefore\ \theta = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi \quad (0 \le \theta \le 2\pi)$$

25. Trigonometry, EXT1 T3 SM-Bank 1



MARKER'S COMMENT: Answers that included a diagram and clearly labelled angles were generally successful.

Show $\theta = \tan^{-1}\left[\dfrac{ax}{x^2 + h(a+h)}\right]$

$\tan A = \dfrac{a+h}{x}$

$\tan B = \dfrac{h}{x}$

$$\begin{aligned}
\tan(A - B) &= \frac{\frac{a+h}{x} - \frac{h}{x}}{1 + \left(\frac{a+h}{x}\right)\left(\frac{h}{x}\right)} \times \frac{x^2}{x^2} \\
&= \frac{x(a+h) - xh}{x^2 + h(a+h)} \\
&= \frac{ax}{x^2 + h(a+h)}
\end{aligned}$$

Since $\theta = A - B$

$\theta = \tan^{-1}\left[\dfrac{ax}{x^2 + h(a+h)}\right]$ ... as required.

26. Trigonometry, EXT1 T1 SM-Bank 3

Base angle: $\sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

$\sin \Rightarrow$ – ve in 4th quadrant

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

Base angle: $\cos\dfrac{\pi}{3} = \dfrac{1}{2}$

$\cos \Rightarrow$ – ve in 2nd quadrant

$$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\begin{aligned}\therefore \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \cos^{-1}\left(-\frac{1}{2}\right) \\ = \frac{\pi}{3} - \left(-\frac{\pi}{3}\right) - \frac{2\pi}{3} \\ = 0\end{aligned}$$