Thank you for subscribing to SmarterMaths Teacher Edition (Silver) in 2022.

Key features of the "2022 HSC Comprehensive Revision Series":

- ~13 hours of cherry picked HSC revision questions by topic
- Weighting toward more difficult examples
- Targeted at motivated students aiming for a Band 5 or 6 result
- ***Attempt, carefully review and annotate*** this revision set in Term 3
- This question set provides the foundation of a concise and high quality revision resource for the run into the HSC exam.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create this revision set that ensures students cover a wide cross-section of the key areas.

IMPORTANT: Exposure to quality HSC questions multiple times is best practice revision and highly beneficial. Many top performing students attest to the benefits of doing quality questions 2-3 times before the HSC. The resulting confidence and *speed through the exam* creates a virtuous cycle for peak performance.

HSC Final Study: ADVANCED MATHEMATICS

F2 Graphing (~4% historical contribution)

Key Areas addressed by this worksheet

Transformations

- it is our view that transformations will be examined like clockwork going forward. This will be in a non-trig context (covered here) or relate to trig function transformations (*covered in the T3 Trig Graphs/Applications*);
- we review the 2021 Adv exam example that proved challenging and produced a mean mark of 48%, along with the 2020 multiple choice question that represents the low end of the difficulty spectrum;
- numerous other higher difficulty examples, informed by the syllabus, NESA sample HSC exam and Topic Guidance, make up this revision set;
- featured function types include quadratics, absolute value and log/exponentials;
- transformation examples look at both algebraic manipulation and subsequent graphing;
- odd functions within this context are covered.

Non-Calculus Graphing

- quotient functions are the most common question type and are a focus area of this revision (last tested in 2021);
- numerous examples require students to find both horizontal and vertical asymptotes;
- manipulating algebraic functions to make graphing easier and more efficient is covered;
- similarly, questions where identifying graphs as odd or even in order to make graphing more efficient are reviewed.

ADVANCED MATHS:
2022 Comprehensive Revision Series
- FUNCTIONS

F2 Graphing
Transformations
Non-Calculus Graphing

Exam Equivalent Time: 60 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

Questions

1. Functions, 2ADV F2 SM-Bank 9 MC

The graph of the function $f(x) = \dfrac{3x + 2}{5 - x}$, has asymptotes at

A. $x = -5, y = \dfrac{3}{2}$

B. $x = \dfrac{2}{3}, y = -3$

C. $x = 5, y = 3$

D. $x = 5, y = -3$

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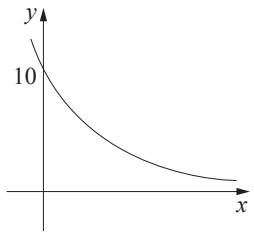
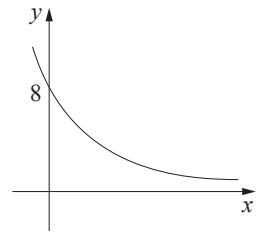
2. Functions, 2ADV F2 2021 HSC 5 MC

Which of the following best represents the graph of $y = 10(0.8)^x$?

A.



B.



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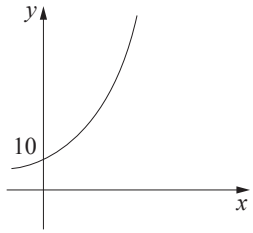
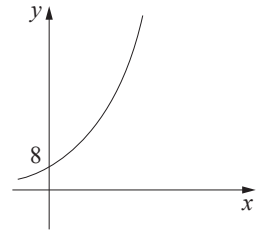


D.



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3. Functions, 2ADV F2 SM-Bank 4 MC

The graph of the function $f(x) = 3x^{\frac{5}{2}}$ is reflected in the x-axis and then translated 3 units to the right and 4 units down.

The equation of the new graph is

A. $y = 3(x - 3)^{\frac{5}{2}} + 4$

B. $y = -3(x - 3)^{\frac{5}{2}} - 4$

C. $y = -3(x + 3)^{\frac{5}{2}} - 1$

D. $y = -3(x - 4)^{\frac{5}{2}} + 3$

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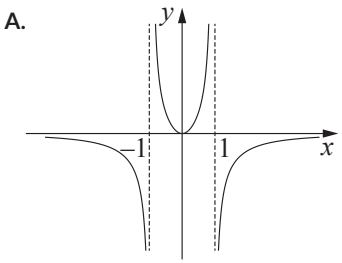
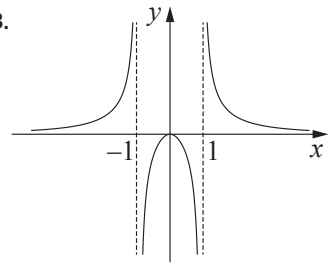
4. Functions, 2ADV' F2 2017 HSC 5 MC

Which graph best represents the function $y = \dfrac{2x^2}{1 - x^2}$?

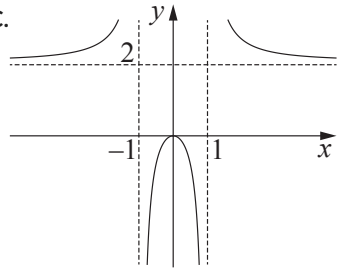
A.



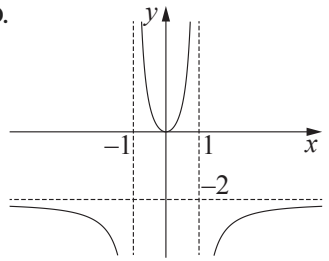
B.



C.



D.



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5. Functions, 2ADV F2 SM-Bank 8 MC

The transformation that maps the graph of $y = \sqrt{8x^3 + 1}$ onto the graph of $y = \sqrt{x^3 + 1}$ is a

A. dilation by a factor of 2 from the y-axis.

B. dilation by a factor of 2 from the x-axis.

C. dilation by a factor of $\dfrac{1}{2}$ from the x-axis.

D. dilation by a factor of $\dfrac{1}{2}$ from the y-axis.

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6. Functions, 2ADV F2 2006 HSC 1c

Sketch the graph of $y = |x + 4|$. *(2 marks)*

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7. Functions, 2ADV F2 SM-Bank 1

i. Draw the graph $y = \ln x$. *(1 mark)*

ii. Explain how the above graph can be transformed to produce the graph

$$y = 3\ln(x + 2)$$

and sketch the graph, clearly identifying all intercepts. *(3 marks)*

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8. Functions, 2ADV F1 SM-Bank 35

i. Sketch the function $y = f(x)$ where $f(x) = (x - 1)^3$ on a number plane, labelling all intercepts. *(1 mark)*

ii. On the same graph, sketch $y = -f(-x)$. Label all intercepts. *(2 marks)*

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9. Functions, 2ADV F2 EQ-Bank 9

$y = -\dfrac{(x + 2)^4}{3}$ has been produced by three successive transformations: a translation, a dilation and then a reflection.

i. Describe each transformation and state the equation of the graph after each transformation. *(2 marks)*

ii. Sketch the graph. *(1 mark)*

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10. Functions, 2ADV F2 2021 HSC 19

Without using calculus, sketch the graph of $y = 2 + \dfrac{1}{x + 4}$, showing the asymptotes and the x and y intercepts. *(3 marks)*

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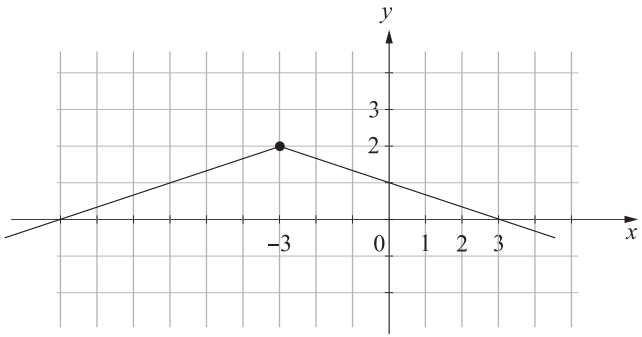
11. Functions, 2ADV F2 EQ-Bank 1

The function $f(x) = |x|$ is transformed and the equation of the new function is $y = kf(x + b) + c$.

The graph of the new function is shown below.



What are the values of k, b and c. *(2 marks)*

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12. Functions, 2ADV F2 SM-Bank 16

Let $f(x) = x^2 - 4$

Let the graph of $g(x)$ be a transformation of the graph of $f(x)$ where the transformations have been applied in the following order:

- dilation by a factor of $\frac{1}{2}$ from the vertical axis (parallel to the horizontal axis)
- translation by two units to the right (in the direction of the positive horizontal axis

Find $g(x)$ and the coordinates of the horizontal axis intercepts of the graph of $g(x)$. *(3 marks)*

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13. Functions, 2ADV F2 SM-Bank 13

i. Show that the function $y = \dfrac{1 - e^x}{1 + e^x}$ is an odd function? *(1 mark)*

ii. Sketch $y = \dfrac{1 - e^x}{1 + e^x}$, labelling all intercepts and asymptotes. *(2 marks)*

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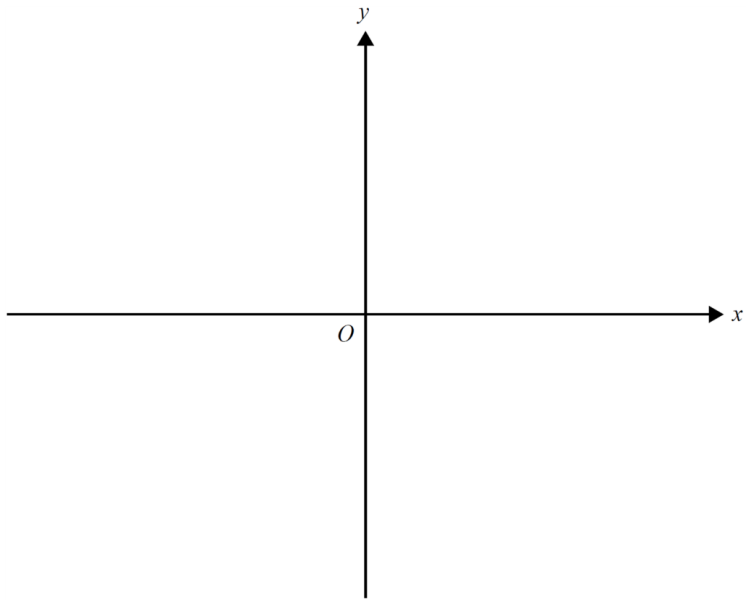
14. Functions, 2ADV F2 EQ-Bank 11

On the axes below, sketch the graph of $f(x) = \dfrac{2x - 2}{x + 1}$.

Label all axis intercepts. Label each asymptote with its equation. *(4 marks)*



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15. Functions, 2ADV F2 EQ-Bank 13

The curve $y = kx^2 + c$ is subject to the following transformations

- Translated 2 units in the positive x-direction
- Dilated in the positive y-direction by a factor of 4
- Reflected in the y-axis

The final equation of the curve is $y = 8x^2 + 32x - 8$.

i. Find the equation of the graph after the dilation. *(1 mark)*

ii. Find the values of k and c. *(2 marks)*

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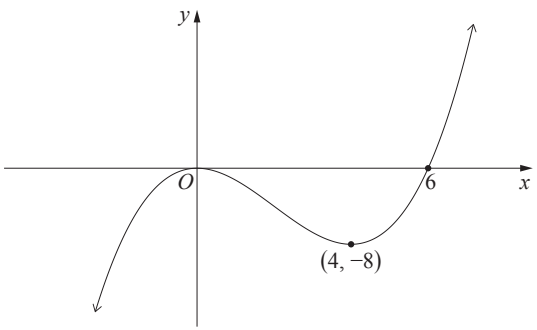
16. Functions, 2ADV F2 2021 HSC 21

Consider the graph of $y = f(x)$ as shown.



NOT TO SCALE

Sketch the graph of $y = 4f(2x)$ showing the x-intercepts and the coordinates of the turning points. *(2 marks)*

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17. Functions, 2ADV F2 2013 HSC 15c

i. Sketch the graph $y = |2x - 3|$. *(1 mark)*

ii. Using the graph from part (i), or otherwise, find all values of m for which the equation $|2x - 3| = mx + 1$ has exactly one solution. *(2 marks)*

Worked Solutions

1. Functions, 2ADV F2 SM-Bank 9 MC

$$f(x) = \frac{3x + 2}{5 - x}$$

$$= \frac{-(15 - 3x) + 17}{5 - x}$$

$$= -3 + \frac{17}{5 - x}$$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = -3$

$\Rightarrow D$

---

2. Functions, 2ADV F2 2021 HSC 5 MC

By elimination:

When $x = 0$, $y = 10(0.8)^0 = 10$

$\rightarrow$ Eliminate B and D

As $x \rightarrow \infty$, $y \rightarrow 0$

$\rightarrow$ Eliminate C

$\Rightarrow A$

---

3. Functions, 2ADV F2 SM-Bank 4 MC

Let $y = 3x^{\frac{5}{2}}$

Reflect in the x-axis:

$$y = -3x^{\frac{5}{2}}$$

Translate 3 units to the right:

$$y = -3(x-3)^{\frac{5}{2}}$$

Translate 4 units down:

$$y = -3(x-3)^{\frac{5}{2}} - 4$$

$\Rightarrow B$

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4. Functions, 2ADV' F2 2017 HSC 5 MC

$$y = \frac{2x^2}{(1-x^2)}$$

$$= -\frac{(2-2x^2-2)}{(1-x^2)}$$

$$= -\frac{2(1-x^2)}{(1-x^2)} - \frac{2}{(1-x^2)}$$

$$= -2 - \frac{2}{(1-x^2)}$$

As $x \to \infty$, $y \to -2$

$\therefore$ Horizontal asymptote at $y = -2$

$\Rightarrow D$

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5. Functions, 2ADV F2 SM-Bank 8 MC

Let $f(x) = \sqrt{8x^3+1}$

$$f\left(\frac{1}{2}x\right) = \sqrt{8\left(\frac{1}{2}x\right)^3 + 1}$$

$$= \sqrt{x^3+1}$$

$\therefore$ Transformation correct when x is swapped for $\frac{x}{2}$

i.e. graph is dilated by factor of 2 from y-axis

$\Rightarrow A$

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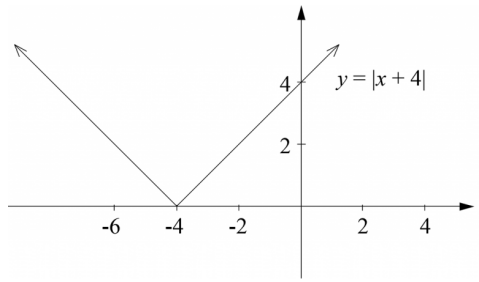
6. Functions, 2ADV F2 2006 HSC 1c



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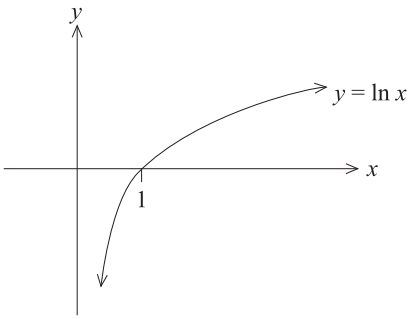
7. Functions, 2ADV F2 SM-Bank 1

i.

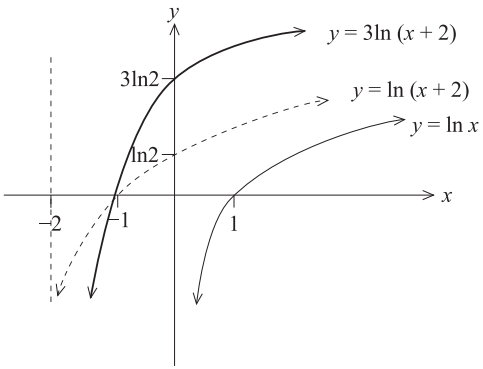


ii. Transforming $y = \ln x \Rightarrow y = \ln(x + 2)$

$y = \ln x \Rightarrow$ shift 2 units to left.

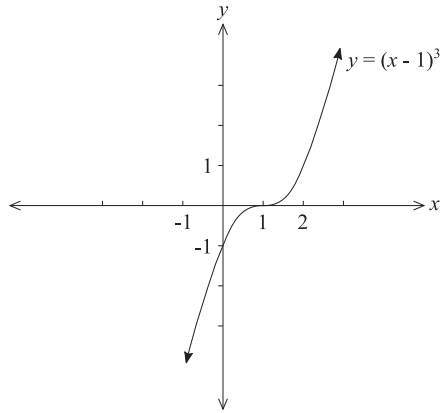
Transforming $y = \ln(x + 2)$ to $y = 3\ln(x + 2)$

$\Rightarrow$ increase each y value by a factor of 3



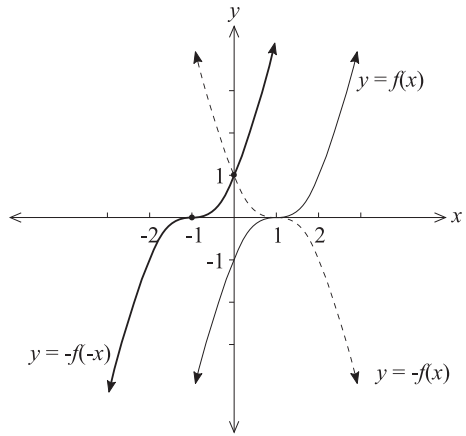
8. Functions, 2ADV F1 SM-Bank 35

i. $y = (x - 1)^3 \Rightarrow y = x^3$ shifted 1 unit to the right.



ii. $y = -f(x) \Rightarrow$ reflect $y = (x - 1)^3$ in x-axis.

$y = -f(-x) \Rightarrow$ reflect $y = -f(x)$ in y-axis.



9. Functions, 2ADV F2 EQ-Bank 9

i. Transformation 1:

Translate $y = x^4$ 2 units to the left.

$y = x^4 \Rightarrow y = (x + 2)^4$

Transformation 2:

Dilate $y = (x + 2)^4$ by a factor of $\frac{1}{3}$ from the x-axis

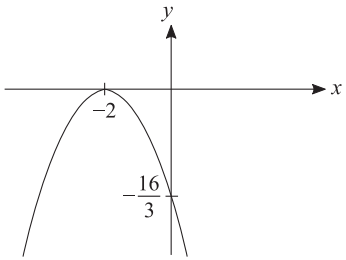
$y = (x + 2)^4 \Rightarrow y = \frac{(x + 2)^4}{3}$

Transformation 3:

Reflect $y = \frac{(x + 2)^4}{3}$ in the x-axis.

$y = \frac{(x + 2)^4}{3} \Rightarrow y = -\frac{(x + 2)^4}{3}$

ii.



10. Functions, 2ADV F2 2021 HSC 19

Asymptotes: $x = -4$

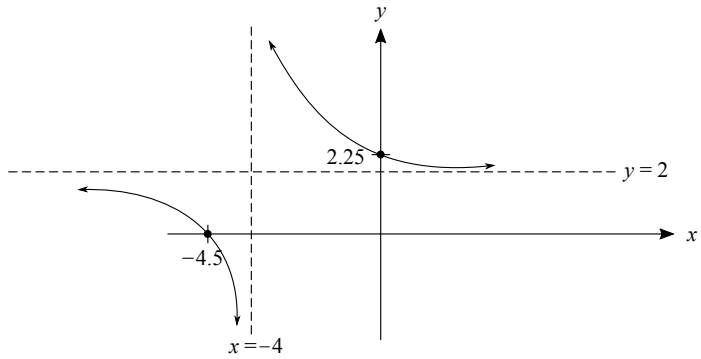
As $x \to \infty, y \to 2$

y-intercept occurs when $x = 0$:

$y = 2.25$

x-intercept occurs when $y = 0$:

$2 + \frac{1}{x + 4} = 0 \Rightarrow x = -4.5$



11. Functions, 2ADV F2 EQ-Bank 1

$y = |x|$

Translate 3 units left $\Rightarrow y = |x + 3|$

Reflect in the x-axis $\Rightarrow y = -|x + 3|$

Dilate by $\frac{1}{3}$ from the x-axis

$\Rightarrow$ Multiply by $\frac{1}{3} \Rightarrow y = -\frac{1}{3}|x + 3|$

Translate 2 units up $\Rightarrow y = -\frac{1}{3}|x + 3| + 2$

$\therefore k = -\frac{1}{3}, b = 3, c = 2$

12. Functions, 2ADV F2 SM-Bank 16

1st transformation

Dilation by a factor of $\frac{1}{2}$ from the y-axis:

$$x^2 - 4 \Rightarrow \left(\frac{x}{\frac{1}{2}}\right)^2 - 4 = 4x^2 - 4$$

2nd transformation

Translation by 2 units to the right:

$4x^2 - 4 \Rightarrow h(x) = 4(x-2)^2 - 4$

x-axis intercept of $h(x)$:

$$4(x-2)^2 - 4 = 0$$
$$(x-2)^2 = 1$$
$$x - 2 = \pm 1$$

$x - 2 = 1 \Rightarrow x = 3$

$x - 2 = -1 \Rightarrow x = 1$

$\therefore$ Horizontal axis intercepts occur at $(1, 0)$ and $(3, 0)$.

13. Functions, 2ADV F2 SM-Bank 13

i. $f(x) = \dfrac{1 - e^x}{1 + e^x}$

$$f(-x) = \frac{1 - e^{-x}}{1 + e^{-x}} \times \frac{e^x}{e^x}$$
$$= \frac{e^x - 1}{e^x + 1}$$
$$= -\frac{1 - e^x}{1 + e^x}$$
$$= -f(x)$$

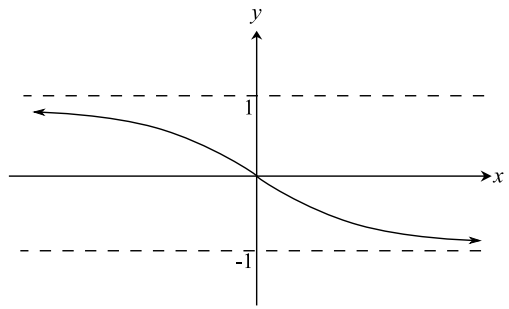
$\therefore f(x)$ is ODD.

ii. $y = \dfrac{1 - e^x}{1 + e^x} \times \dfrac{e^{-x}}{e^{-x}} = \dfrac{e^{-x} - 1}{e^{-x} + 1} = 1 - \dfrac{2}{e^{-x} + 1}$

As $x \to \infty$, $\dfrac{2}{e^{-x} + 1} \to 2$, $y \to -1$

As $x \to -\infty$, $\dfrac{2}{e^{-x} + 1} \to 0$, $y \to 1$

When $x = 0$, $y = 0$



14. Functions, 2ADV F2 EQ-Bank 11

$$\frac{2x-2}{x+1} = \frac{2(x+1)-4}{x+1}$$
$$= 2 - \frac{4}{(x+1)}$$

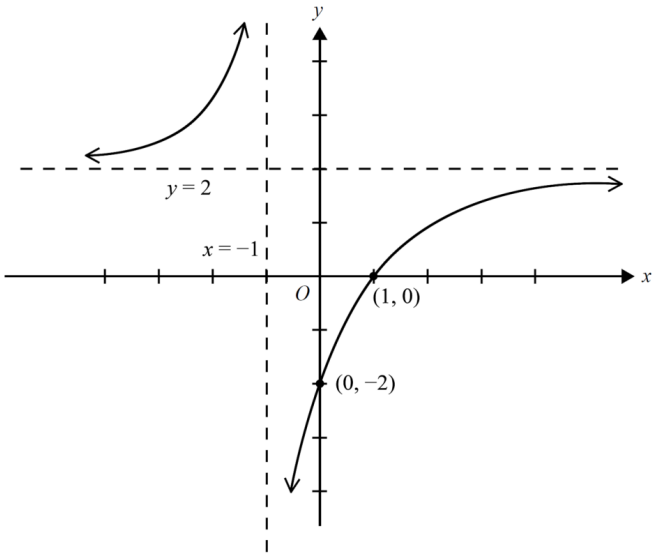
Asymptotes: $x = -1, \quad y = 2$

As $x \to \infty, \quad y \to 2(-)$

As $x \to -\infty, \quad y \to 2(+)$

As $x \to -1(-), \quad y \to \infty$

As $x \to -1(+), \quad y \to -\infty$



15. Functions, 2ADV F2 EQ-Bank 13

i. $y = kx^2 + c$

Translate 2 units in positive x-direction.

$y = kx^2 + c \Rightarrow y = k(x-2)^2 + c$

Dilate in the positive y-direction by a factor of 4.

$y = k(x-2)^2 + c \Rightarrow y = 4k(x-2)^2 + 4c$

ii. $y = 4k(x^2 - 4x + 4) + 4c$

$= 4kx^2 - 16kx + 16k + 4c$

Reflect in the y-axis.

$\Rightarrow$ Swap: $x \to -x$

$y = 4k(-x)^2 - 16k(-x) + 16k + 4c$

$= 4kx^2 + 16kx + 16k + 4c$

COMMENT: Using "swap" terminology for reflections in the y-axis is simpler and more intelligible for students in our view.

Equating co-efficients:

$4k = 8$

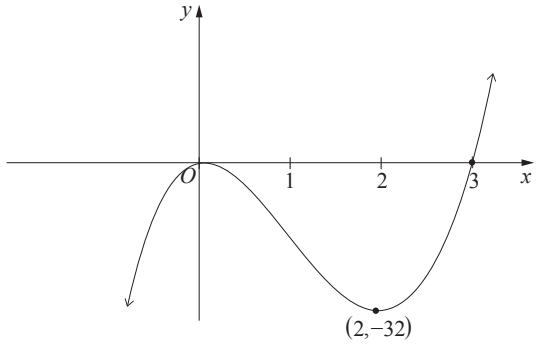
$\therefore k = 2$

$16k + 4c = -8$

$4c = -40$

$\therefore c = -10$

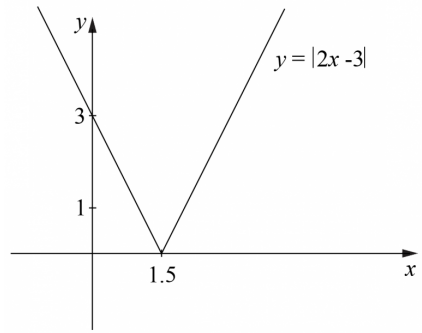
16. Functions, 2ADV F2 2021 HSC 21



♦ *Mean mark 48%.*

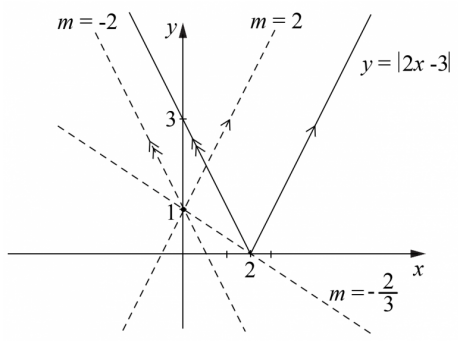
17. Functions, 2ADV F2 2013 HSC 15c

i.



♦ *Mean mark 49%*
MARKER'S COMMENT: Many students drew diagrams that were "too small", didn't use rulers or didn't use a consistent scale on the axes!

ii.



Line of intersection $y = mx + 1$ passes through $(0, 1)$

If it also passes through $(1.5, 0) \Rightarrow$ 1 solution

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{1 - 0}{0 - \frac{3}{2}}$$
$$= -\frac{2}{3}$$

Gradients of $y = |2x - 3|$ are 2 or -2

Considering a line through $(0, 1)$:

♦♦ *Mean mark 25%.*
COMMENT: Students need a clear graphical understanding of what they are finding to solve this very challenging, Band 6 question.

If $m \geq 2$, only intersects once.

Similarly,

If $m < -2$, only intersects once.

$\therefore$ Only one solution when $m = -\frac{2}{3}$, $m \geq 2$ or $m < -2$