

ADVANCED MATHS:
2022 Comprehensive Revision Series

- FUNCTIONS

F2 Graphing
Transformations
Non-Calculus Graphing

Exam Equivalent Time: 60 minutes (based on HSC allocation of 1.5 minutes approx. per mark)



Questions

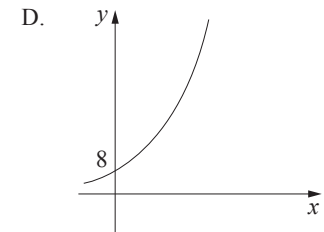
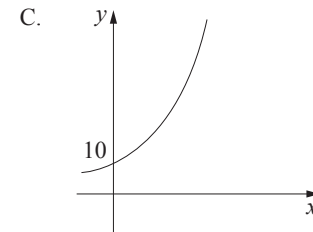
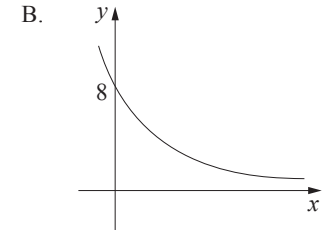
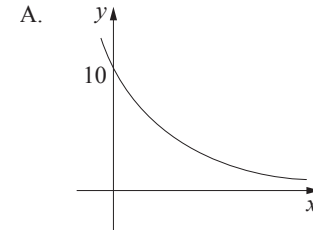
1. Functions, 2ADV F2 SM-Bank 9 MC

The graph of the function $f(x) = \frac{3x + 2}{5 - x}$, has asymptotes at

- A. $x = -5, y = \frac{3}{2}$
- B. $x = \frac{2}{3}, y = -3$
- C. $x = 5, y = 3$
- D. $x = 5, y = -3$

2. Functions, 2ADV F2 2021 HSC 5 MC

Which of the following best represents the graph of $y = 10(0.8)^x$?



3. Functions, 2ADV F2 SM-Bank 4 MC

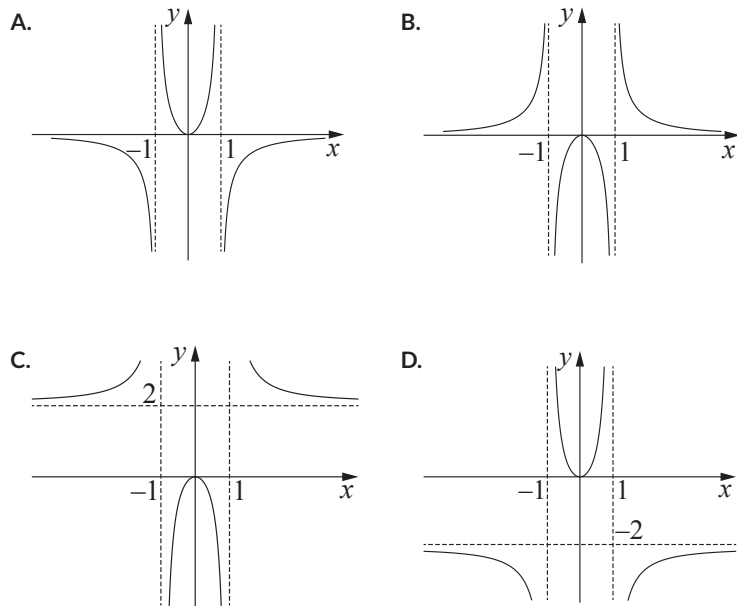
The graph of the function $f(x) = 3x^{\frac{5}{2}}$ is reflected in the x -axis and then translated 3 units to the right and 4 units down.

The equation of the new graph is

- A. $y = 3(x - 3)^{\frac{5}{2}} + 4$
- B. $y = -3(x - 3)^{\frac{5}{2}} - 4$
- C. $y = -3(x + 3)^{\frac{5}{2}} - 1$
- D. $y = -3(x - 4)^{\frac{5}{2}} + 3$

4. Functions, 2ADV F2 2017 HSC 5 MC

Which graph best represents the function $y = \frac{2x^2}{1-x^2}$?



5. Functions, 2ADV F2 SM-Bank 8 MC

The transformation that maps the graph of $y = \sqrt{8x^3 + 1}$ onto the graph of $y = \sqrt{x^3 + 1}$ is a

- dilation by a factor of 2 from the y -axis.
- dilation by a factor of 2 from the x -axis.
- dilation by a factor of $\frac{1}{2}$ from the x -axis.
- dilation by a factor of $\frac{1}{2}$ from the y -axis.

6. Functions, 2ADV F2 2006 HSC 1c

Sketch the graph of $y = |x + 4|$. (2 marks)

7. Functions, 2ADV F2 SM-Bank 1

- Draw the graph $y = \ln x$. (1 mark)
- Explain how the above graph can be transformed to produce the graph

$$y = 3 \ln(x + 2)$$

and sketch the graph, clearly identifying all intercepts. (3 marks)

8. Functions, 2ADV F1 SM-Bank 35

- Sketch the function $y = f(x)$ where $f(x) = (x - 1)^3$ on a number plane, labelling all intercepts. (1 mark)
- On the same graph, sketch $y = -f(-x)$. Label all intercepts. (2 marks)

9. Functions, 2ADV F2 EQ-Bank 9

$y = -\frac{(x + 2)^4}{3}$ has been produced by three successive transformations: a translation, a dilation and then a reflection.

- Describe each transformation and state the equation of the graph after each transformation. (2 marks)
- Sketch the graph. (1 mark)

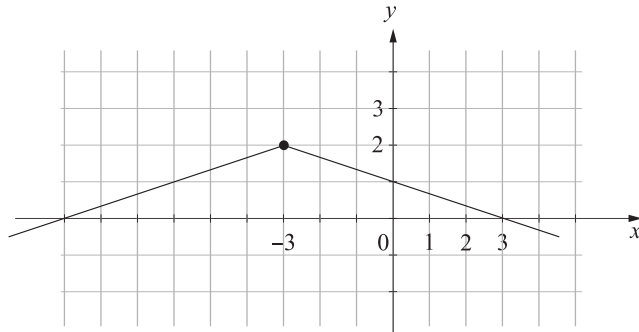
10. Functions, 2ADV F2 2021 HSC 19

Without using calculus, sketch the graph of $y = 2 + \frac{1}{x + 4}$, showing the asymptotes and the x and y intercepts. (3 marks)

11. Functions, 2ADV F2 EQ-Bank 1

The function $f(x) = |x|$ is transformed and the equation of the new function is $y = kf(x + b) + c$.

The graph of the new function is shown below.



What are the values of k , b and c . (2 marks)

12. Functions, 2ADV F2 SM-Bank 16

Let $f(x) = x^2 - 4$

Let the graph of $g(x)$ be a transformation of the graph of $f(x)$ where the transformations have been applied in the following order:

- dilation by a factor of $\frac{1}{2}$ from the vertical axis (parallel to the horizontal axis)
- translation by two units to the right (in the direction of the positive horizontal axis)

Find $g(x)$ and the coordinates of the horizontal axis intercepts of the graph of $g(x)$. (3 marks)

13. Functions, 2ADV F2 SM-Bank 13

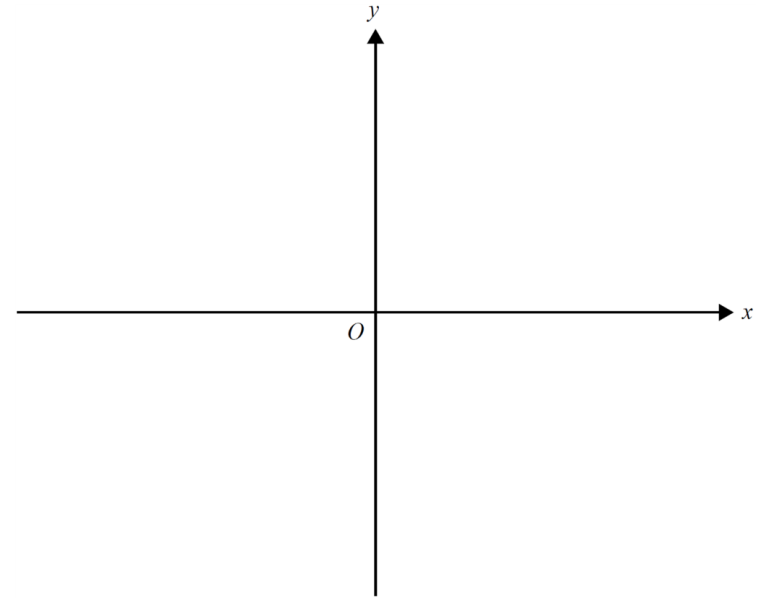
i. Show that the function $y = \frac{1 - e^x}{1 + e^x}$ is an odd function? (1 mark)

ii. Sketch $y = \frac{1 - e^x}{1 + e^x}$, labelling all intercepts and asymptotes. (2 marks)

14. Functions, 2ADV F2 EQ-Bank 11

On the axes below, sketch the graph of $f(x) = \frac{2x - 2}{x + 1}$.

Label all axis intercepts. Label each asymptote with its equation. (4 marks)



15. Functions, 2ADV F2 EQ-Bank 13

The curve $y = kx^2 + c$ is subject to the following transformations

- Translated 2 units in the positive x -direction
- Dilated in the positive y -direction by a factor of 4
- Reflected in the y -axis

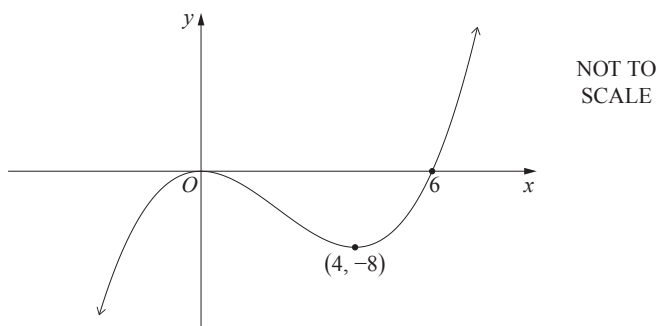
The final equation of the curve is $y = 8x^2 + 32x - 8$.

i. Find the equation of the graph after the dilation. (1 mark)

ii. Find the values of k and c . (2 marks)

16. Functions, 2ADV F2 2021 HSC 21

Consider the graph of $y = f(x)$ as shown.



Sketch the graph of $y = 4f(2x)$ showing the x -intercepts and the coordinates of the turning points. (2 marks)

17. Functions, 2ADV F2 2013 HSC 15c

i. Sketch the graph $y = |2x - 3|$. (1 mark)

ii. Using the graph from part (i), or otherwise, find all values of m for which the equation $|2x - 3| = mx + 1$ has exactly one solution. (2 marks)

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Worked Solutions

1. Functions, 2ADV F2 SM-Bank 9 MC

$$\begin{aligned} f(x) &= \frac{3x + 2}{5 - x} \\ &= \frac{-(15 - 3x) + 17}{5 - x} \\ &= -3 + \frac{17}{5 - x} \end{aligned}$$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = -3$

$\Rightarrow D$

2. Functions, 2ADV F2 2021 HSC 5 MC

By elimination:

When $x = 0$, $y = 10(0.8)^0 = 10$

\rightarrow Eliminate B and D

As $x \rightarrow \infty$, $y \rightarrow 0$

\rightarrow Eliminate C

$\Rightarrow A$

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3. Functions, 2ADV F2 SM-Bank 4 MC

$$\text{Let } y = 3x^{\frac{5}{2}}$$

Reflect in the x -axis:

$$y = -3x^{\frac{5}{2}}$$

Translate 3 units to the right:

$$y = -3(x - 3)^{\frac{5}{2}}$$

Translate 4 units down:

$$y = -3(x - 3)^{\frac{5}{2}} - 4$$

$\Rightarrow B$

4. Functions, 2ADV' F2 2017 HSC 5 MC

$$\begin{aligned} y &= \frac{2x^2}{(1-x^2)} \\ &= -\frac{(2-2x^2-2)}{(1-x^2)} \\ &= -\frac{2(1-x^2)}{(1-x^2)} - \frac{2}{(1-x^2)} \\ &= -2 - \frac{2}{(1-x^2)} \end{aligned}$$

As $x \rightarrow \infty$, $y \rightarrow -2$

\therefore Horizontal asymptote at $y = -2$

$\Rightarrow D$

5. Functions, 2ADV F2 SM-Bank 8 MC

$$\text{Let } f(x) = \sqrt{8x^3 + 1}$$

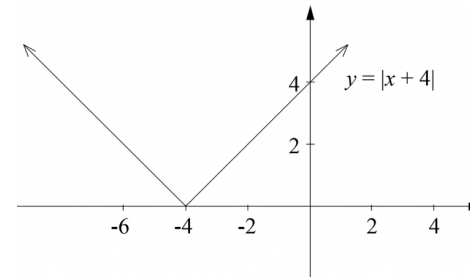
$$\begin{aligned} f\left(\frac{1}{2}x\right) &= \sqrt{8\left(\frac{1}{2}x\right)^3 + 1} \\ &= \sqrt{x^3 + 1} \end{aligned}$$

\therefore Transformation correct when x is swapped for $\frac{x}{2}$

i.e. graph is dilated by factor of 2 from y -axis

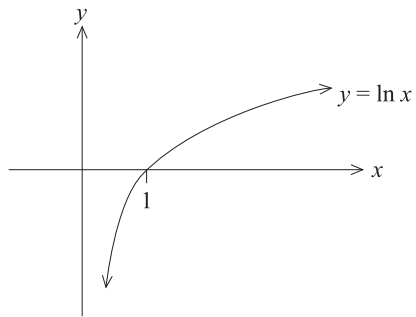
$\Rightarrow A$

6. Functions, 2ADV F2 2006 HSC 1c



7. Functions, 2ADV F2 SM-Bank 1

i.

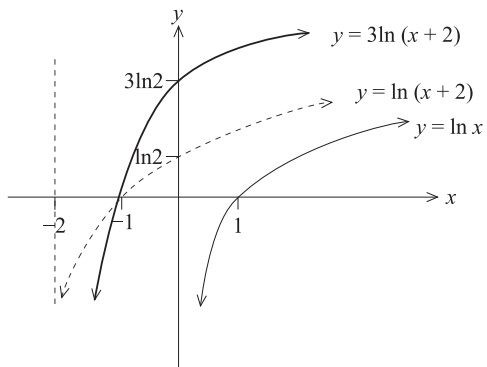


ii. Transforming $y = \ln x \Rightarrow y = \ln(x + 2)$

$y = \ln x \Rightarrow$ shift 2 units to left.

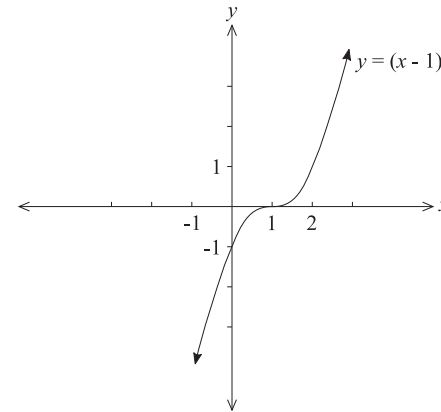
Transforming $y = \ln(x + 2)$ to $y = 3 \ln(x + 2)$

\Rightarrow increase each y value by a factor of 3



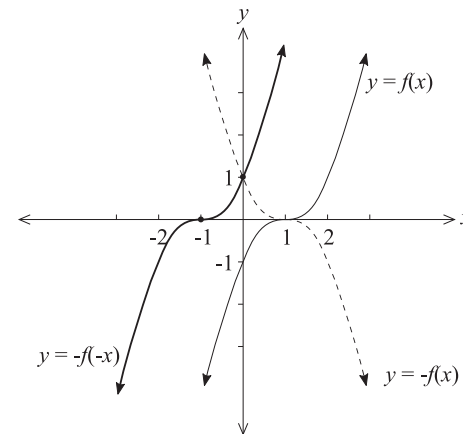
8. Functions, 2ADV F1 SM-Bank 35

i. $y = (x - 1)^3 \Rightarrow y = x^3$ shifted 1 unit to the right.



ii. $y = -f(x) \Rightarrow$ reflect $y = (x - 1)^3$ in x -axis.

$y = -f(-x) \Rightarrow$ reflect $y = -f(x)$ in y -axis.



9. Functions, 2ADV F2 EQ-Bank 9

i. Transformation 1:

Translate $y = x^4$ 2 units to the left.

$$y = x^4 \Rightarrow y = (x + 2)^4$$

Transformation 2:

Dilate $y = (x + 2)^4$ by a factor of $\frac{1}{3}$ from the x -axis

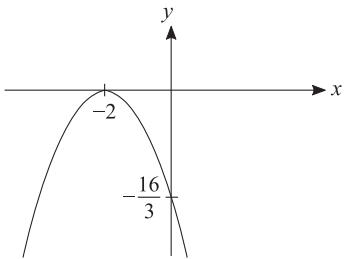
$$y = (x + 2)^4 \Rightarrow y = \frac{(x + 2)^4}{3}$$

Transformation 3:

Reflect $y = \frac{(x + 2)^4}{3}$ in the x -axis.

$$y = \frac{(x + 2)^4}{3} \Rightarrow y = -\frac{(x + 2)^4}{3}$$

ii.



10. Functions, 2ADV F2 2021 HSC 19

Asymptotes: $x = -4$

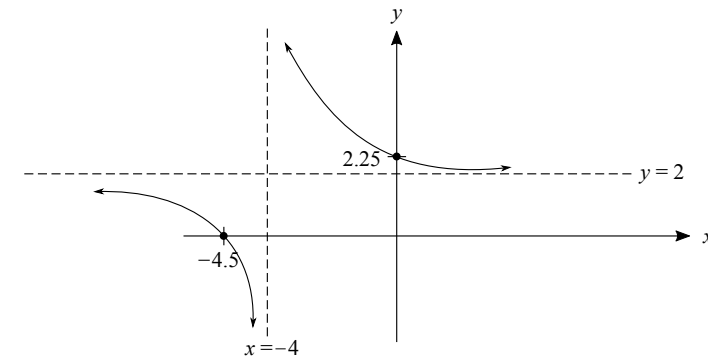
As $x \rightarrow \infty, y \rightarrow 2$

y -intercept occurs when $x = 0$:

$$y = 2.25$$

x -intercept occurs when $y = 0$:

$$2 + \frac{1}{x + 4} = 0 \Rightarrow x = -4.5$$



11. Functions, 2ADV F2 EQ-Bank 1

$$y = |x|$$

Translate 3 units left $\Rightarrow y = |x + 3|$

Reflect in the x -axis $\Rightarrow y = -|x + 3|$

Dilate by $\frac{1}{3}$ from the x -axis

$$\Rightarrow \text{Multiply by } \frac{1}{3} \Rightarrow y = -\frac{1}{3}|x + 3|$$

Translate 2 units up $\Rightarrow y = -\frac{1}{3}|x + 3| + 2$

$$\therefore k = -\frac{1}{3}, b = 3, c = 2$$

12. Functions, 2ADV F2 SM-Bank 16

1st transformation

Dilation by a factor of $\frac{1}{2}$ from the y-axis:

$$x^2 - 4 \Rightarrow \left(\frac{x}{\frac{1}{2}}\right)^2 - 4 = 4x^2 - 4$$

2nd transformation

Translation by 2 units to the right:

$$4x^2 - 4 \Rightarrow h(x) = 4(x - 2)^2 - 4$$

x-axis intercept of $h(x)$:

$$4(x - 2)^2 - 4 = 0$$

$$(x - 2)^2 = 1$$

$$x - 2 = \pm 1$$

$$x - 2 = 1 \Rightarrow x = 3$$

$$x - 2 = -1 \Rightarrow x = 1$$

\therefore Horizontal axis intercepts occur at (1, 0) and (3, 0).

13. Functions, 2ADV F2 SM-Bank 13

$$\text{i. } f(x) = \frac{1 - e^x}{1 + e^x}$$

$$\begin{aligned} f(-x) &= \frac{1 - e^{-x}}{1 + e^{-x}} \times \frac{e^x}{e^x} \\ &= \frac{e^x - 1}{e^x + 1} \\ &= -\frac{1 - e^x}{1 + e^x} \\ &= -f(x) \end{aligned}$$

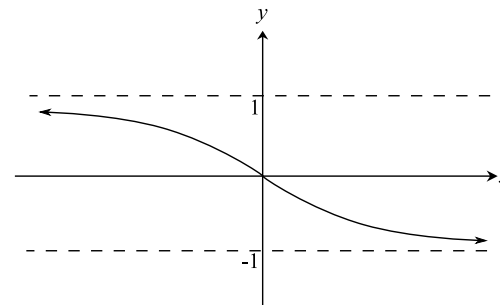
$\therefore f(x)$ is ODD.

$$\text{ii. } y = \frac{1 - e^x}{1 + e^x} \times \frac{e^{-x}}{e^{-x}} = \frac{e^{-x} - 1}{e^{-x} + 1} = 1 - \frac{2}{e^{-x} + 1}$$

$$\text{As } x \rightarrow \infty, \frac{2}{e^{-x} + 1} \rightarrow 2, y \rightarrow -1$$

$$\text{As } x \rightarrow -\infty, \frac{2}{e^{-x} + 1} \rightarrow 0, y \rightarrow 1$$

When $x = 0$, $y = 0$



14. Functions, 2ADV F2 EQ-Bank 11

$$\begin{aligned}\frac{2x-2}{x+1} &= \frac{2(x+1)-4}{x+1} \\ &= 2 - \frac{4}{x+1}\end{aligned}$$

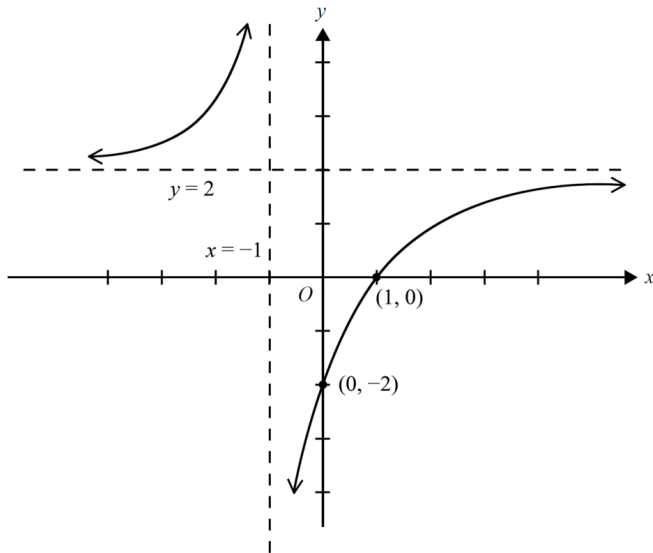
Asymptotes: $x = -1$, $y = 2$

As $x \rightarrow \infty$, $y \rightarrow 2(-)$

As $x \rightarrow -\infty$, $y \rightarrow 2(+)$

As $x \rightarrow -1(-)$, $y \rightarrow \infty$

As $x \rightarrow -1(+)$, $y \rightarrow -\infty$



15. Functions, 2ADV F2 EQ-Bank 13

i. $y = kx^2 + c$

Translate 2 units in positive x -direction.

$$y = kx^2 + c \Rightarrow y = k(x-2)^2 + c$$

Dilate in the positive y -direction by a factor of 4.

$$y = k(x-2)^2 + c \Rightarrow y = 4k(x-2)^2 + 4c$$

ii. $y = 4k(x^2 - 4x + 4) + 4c$
 $= 4kx^2 - 16kx + 16k + 4c$

Reflect in the y -axis.

$$\Rightarrow \text{Swap: } x \rightarrow -x$$

$$\begin{aligned}y &= 4k(-x)^2 - 16k(-x) + 16k + 4c \\ &= 4kx^2 + 16kx + 16k + 4c\end{aligned}$$

Equating co-efficients:

$$4k = 8$$

$$\therefore k = 2$$

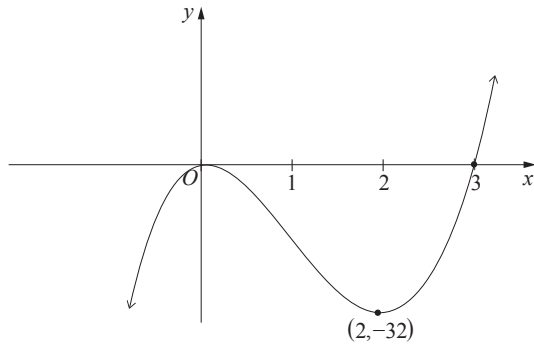
$$16k + 4c = -8$$

$$4c = -40$$

$$\therefore c = -10$$

COMMENT: Using "swap" terminology for reflections in the y -axis is simpler and more intelligible for students in our view.

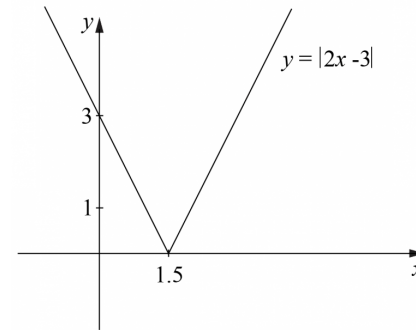
16. Functions, 2ADV F2 2021 HSC 21



♦ Mean mark 48%.

17. Functions, 2ADV F2 2013 HSC 15c

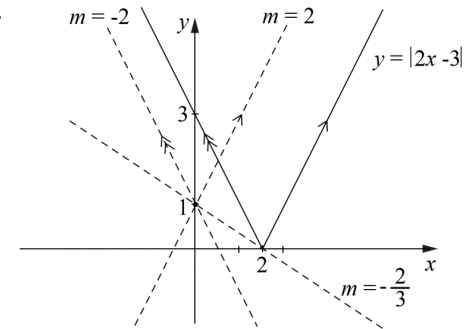
i.



♦ Mean mark 49%

MARKER'S COMMENT: Many students drew diagrams that were "too small", didn't use rulers or didn't use a consistent scale on the axes!

ii.



Line of intersection $y = mx + 1$ passes through $(0, 1)$

If it also passes through $(1.5, 0) \Rightarrow 1$ solution

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - 0}{0 - \frac{3}{2}} \\ &= -\frac{2}{3} \end{aligned}$$

Gradients of $y = |2x - 3|$ are 2 or -2

Considering a line through $(0, 1)$:

♦♦ Mean mark 25%.

COMMENT: Students need a clear graphical understanding of what they are finding to solve this very challenging, Band 6 question.

If $m \geq 2$, only intersects once.

Similarly,

If $m < -2$, only intersects once.

\therefore Only one solution when $m = -\frac{2}{3}$, $m \geq 2$ or $m < -2$