Thank you for subscribing to SmarterMaths Teacher Edition (Silver) in 2022.

Key features of the "2022 HSC Comprehensive Revision Series":

- ~13 hours of cherry picked HSC revision questions by topic
- Weighting toward more difficult examples
- Targeted at motivated students aiming for a Band 5 or 6 result
- ***Attempt, carefully review and annotate*** this revision set in Term 3
- This question set provides the foundation of a concise and high quality revision resource for the run into the HSC exam.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create this revision set that ensures students cover a wide cross-section of the key areas.

IMPORTANT: Exposure to quality HSC questions multiple times is best practice revision and highly beneficial. Many top performing students attest to the benefits of doing quality questions 2-3 times before the HSC. The resulting confidence and *speed through the exam* creates a virtuous cycle for peak performance.

HSC Final Study: EXTENSION 1 MATHEMATICS

VECTORS: V1 Introduction to Vectors

Key Areas addressed by this worksheet

Operations With Vectors

- the *2020-21 Ext1* exams both included 3 separate questions in this topic area. It represents critical revision which is reflected in this worksheet;
- basic vector calculations, angles between vectors, unit vectors and projections are all well covered. A graphical interpretation of a projection was required in 2020 that warrants attention;
- solving problems using the equivalence of the two scalar dot product formulae was not examined in 2020-21. We regard this as an important question type and multiple examples look at this concept;
- we note that column vector notation dominates the worked solutions. In our view, this helps students visualise calculations and minimise errors.

Vectors, Force and Velocity

- we review the important 2021 question that looked at vectors within a motion theme, along with multiple questions looking at vectors and force;
- questions modelled off the NESA Ext1 sample exam are reviewed.

Vectors and Geometry

- a challenging area allocated 4-marks in the 2021 exam and 1 mark in 2020;
- questions from a wide range of difficulty levels are covered. We note the 2021 example produced a mean mark of just 18% and deserves close attention;
- a number of geometric proofs specifically mentioned in the syllabus are reviewed;
- other geometric proofs are revised, with NESA sample HSC and exemplar questions providing direction for these selections.

Vectors and Projectile Motion

- this topic area, when examined, will likely involve a high level of difficulty and a significant mark allocation, as evidenced by the 2021 exam's 4-mark question;
- our revision focus is on the parametric vector equations of motion (functions of t). A student's conceptual understanding of maximum height, time of flight, range etc.. are all key concepts reviewed;
- we note the NESA sample exam involved the Cartesian equation of motion (y as a function of x) and this is also reviewed;
- NESA exemplar questions borrow from past VCE exams and we have used this direction to select other similar VCE questions (© approved).

EXTENSION 1 MATHS:
2022 Comprehensive Revision Series
- VECTORS

V1 Introduction to Vectors
Operations With Vectors
Vectors, Force and Velocity
Vectors and Geometry
Vectors and Projectile Motion

Exam Equivalent Time: ~120 minutes *(based on HSC allocation of 1.7 minutes approx. per mark)*

Questions

1. Vectors, EXT1 V1 SM-Bank 16 MC

The vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + m\underset{\sim}{j}$ and $\underset{\sim}{b} = m^2\underset{\sim}{i} - \underset{\sim}{j}$ are perpendicular for

A. $m = -2$ and $m = 0$

B. $m = 2$ and $m = 0$

C. $m = -\dfrac{1}{2}$ and $m = 0$

D. $m = \dfrac{1}{2}$ and $m = 0$

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2. Vectors, EXT1 V1 2020 HSC 4 MC

Maria starts at the origin and walks along all of the vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$, then walks along all of the vector $3\underset{\sim}{i} - 2\underset{\sim}{j}$ and finally along all of the vector $4\underset{\sim}{i} - 3\underset{\sim}{j}$.

How far from the origin is she?

A. $\sqrt{77}$

B. $\sqrt{85}$

C. $2\sqrt{13} + \sqrt{5}$

D. $\sqrt{5} + \sqrt{7} + \sqrt{13}$

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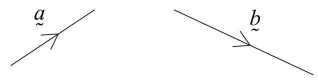
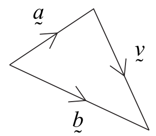
3. Vectors, EXT1 V1 2020 HSC 6 MC

The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are shown.

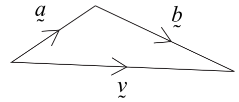


Which diagram below shows the vector $\underset{\sim}{v} = \underset{\sim}{a} - \underset{\sim}{b}$?

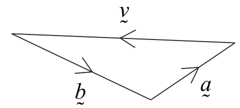
A.



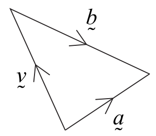
B.



C.



D.



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4. Vectors, EXT1 V1 2021 HSC 1 MC

Given that $\overrightarrow{OP} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, what is $\overrightarrow{PQ}$?

A. $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$

B. $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$

C. $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$

D. $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$

---

5. Vectors, EXT1 V1 2021 HSC 5 MC

For the two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ it is know that

$$\overrightarrow{OA} \cdot \overrightarrow{OB} < 0$$

Which of the following statements MUST be true?

A. Either, $\overrightarrow{OA}$ is negative and $\overrightarrow{OB}$ is positive, or $\overrightarrow{OA}$ is positive and $\overrightarrow{OB}$ is negative.

B. The angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is obtuse.

C. The product $\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|$ is negative.

D. The points O, A and B are collinear.

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6. Vectors, EXT1 V1 2020 HSC 9 MC

The projection of the vector $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$ onto the line $y = 2x$ is $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$.

The point $(6, 7)$ is reflected in the line $y = 2x$ to a point A.

What is the position vector of the point A?

A. $\begin{pmatrix} 6 \\ 12 \end{pmatrix}$

B. $\begin{pmatrix} 2 \\ 9 \end{pmatrix}$

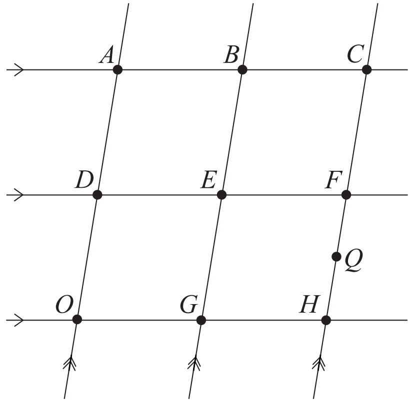
C. $\begin{pmatrix} -6 \\ 7 \end{pmatrix}$

D. $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

---

7. Vectors, EXT1 V1 EQ-Bank 4 MC

The diagram shows a grid of equally spaced lines. The vector $\overrightarrow{OA} = \underset{\sim}{a}$ and the vector $\overrightarrow{OH} = \underset{\sim}{h}$. The point Q is halfway between F and H.



Which expression represents the vector $\overrightarrow{EQ}$?

A. $-\frac{1}{4}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{h}$

B. $\frac{1}{2}\underset{\sim}{a} - \frac{1}{4}\underset{\sim}{h}$

C. $\frac{1}{4}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{h}$

D. $\frac{1}{4}\underset{\sim}{a} + \underset{\sim}{h}$

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8. Vectors, EXT1 V1 2020 HSC 11b

For what values(s) of a are the vectors $\begin{pmatrix} a \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2a-3 \\ 2 \end{pmatrix}$ perpendicular? *(3 marks)*

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9. Vectors, EXT1 V1 EQ-Bank 4

Let the vectors $\underset{\sim}{a} = 4\underset{\sim}{i} - \underset{\sim}{j}$, $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{c} = -2\underset{\sim}{i} + 5\underset{\sim}{j}$.

i. Calculate $\underset{\sim}{a} \cdot \left(\underset{\sim}{b} + \underset{\sim}{c}\right)$ *(1 mark)*

ii. Verify $\underset{\sim}{a} \cdot \left(\underset{\sim}{b} + \underset{\sim}{c}\right) = \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{c}$ *(1 mark)*

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10. Vectors, EXT1 V1 EQ-Bank 7

The vectors $\underset{\sim}{a} = 6\underset{\sim}{i} + 2\underset{\sim}{j}$, $\underset{\sim}{b} = \underset{\sim}{i} - 5\underset{\sim}{j}$ and $\underset{\sim}{c} = 4\underset{\sim}{i} + 4\underset{\sim}{j}$

Find the values of m and n such that $m\underset{\sim}{a} + n\underset{\sim}{b} = \underset{\sim}{c}$. *(2 marks)*

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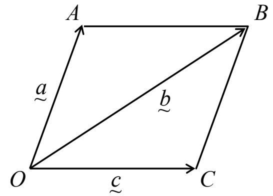
11. Vectors, EXT1 V1 EQ-Bank 27



$OABC$ are the vertices of a rhombus.

Prove, using vectors, that its diagonals are perpendicular. *(2 marks)*

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12. Vectors, EXT1 V1 SM-Bank 20

Consider the vector $\underset{\sim}{a} = \underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$, where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors in the positive direction of the x and y axes respectively.

i. Find the unit vector in the direction of $\underset{\sim}{a}$. *(1 mark)*

ii. Find the acute angle that $\underset{\sim}{a}$ makes with the positive direction of the x-axis. *(1 mark)*

iii. The vector $\underset{\sim}{b} = m\underset{\sim}{i} - 2\underset{\sim}{j}$.

Given that $\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{a}$, find the value of $\underset{\sim}{m}$. *(1 mark)*

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13. Vectors, EXT1 V1 EQ-Bank 28

A projectile is fired from a canon at ground level with initial velocity $\sqrt{300}$ ms^{-1} at an angle of 30° to the horizontal.

The equations of motion are $\dfrac{d^2x}{dt^2} = 0$ and $\dfrac{d^2y}{dt^2} = -10$.

i. Show that $x = 15t$. *(1 mark)*

ii. Show that $y = 5\sqrt{3}t - 5t^2$. *(2 marks)*

iii. Hence find the Cartesian equation for the trajectory of the projectile. *(1 mark)*

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14. Vectors, EXT1 V1 2021 HSC 13b

When an object is projected from a point h metres above the origin with initial speed V m/s at an angle of $\theta°$ to the horizontal, its displacement vector, t seconds after projection, is

$$\underset{\sim}{r}(t) = (Vt\cos\theta)\underset{\sim}{i} + \left(-5t^2 + Vt\sin\theta + h\right)\underset{\sim}{j}.$$ (Do NOT prove this.)

A person, standing in an empty room which is 3 m high, throws a ball at the far wall of the room. The ball leaves their hand 1 m above the floor and 10 m from the far wall. The initial velocity of the ball is 12 m/s at an angle of 30° to the horizontal.

Show that the ball will NOT hit the ceiling of the room but that it will hit the far wall without hitting the floor. *(4 marks)*

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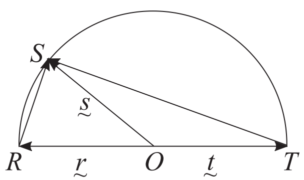
15. Vectors, EXT1 V1 EQ-Bank 2

In the diagram below, ROT is a diameter of the circle with centre O.

S is a point on the circumference.



Using the properties of vectors $\underset{\sim}{r}$, $\underset{\sim}{s}$ and $\underset{\sim}{t}$, show that $\angle RST$ is a right angle. *(2 marks)*

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16. Vectors, EXT1 V1 EQ-Bank 3

A force described by the vector $\underset{\sim}{F} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ newtons is applied to a line l which is parallel to the vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

i. Find the component of the force $\underset{\sim}{F}$ in the direction of l. *(2 marks)*

ii. What is the component of the force $\underset{\sim}{F}$ in the direction perpendicular to the line? *(1 mark)*

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17. Vectors, EXT1 V1 EQ-Bank 5

Using vectors, calculate the acute angle between the line that passes through $A(1, 3)$ and $B(2, -6)$ and the line that passes through $C(1, 5)$ and $D(3, -2)$.

Give your answer correct to one decimal place. *(2 marks)*

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18. Vectors, EXT1 V1 EQ-Bank 6

Point C lies on AB such that $\overrightarrow{AC} = \lambda\overrightarrow{AB}$.

i. Express $\underset{\sim}{c}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. *(2 marks)*

ii. Hence or otherwise, show that $\overrightarrow{BC} = (1-\lambda)\left(\underset{\sim}{a} - \underset{\sim}{b}\right)$. *(1 mark)*

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19. Vectors, EXT1 V1 SM-Bank 13

Given $\underset{\sim}{a} = 4\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 7\underset{\sim}{i} - \underset{\sim}{j}$, what is the magnitude of the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. Give your answer in simplest form. *(3 marks)*

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20. Vectors, EXT1 V1 SM-Bank 18

If θ is the angle between $\underset{\sim}{a} = \underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + \underset{\sim}{j}$, then find the exact value of $\cos 2\theta$. *(2 marks)*

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21. Vectors, EXT1 V1 SM-Bank 21

Relative to a fixed origin, the points A, B and C are defined respectively by the position vectors $\underset{\sim}{a} = -\underset{\sim}{i} - \underset{\sim}{j},\ \underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{c} = -a\underset{\sim}{i} + 2\underset{\sim}{j}$, where a is a real constant.

If the magnitude of angle ABC is $\dfrac{\pi}{3}$, find a. *(3 marks)*

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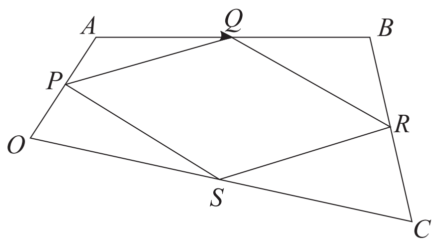
22. Vectors, EXT1 V1 EQ-Bank 26

$OABC$ is a quadrilateral.

P, Q, R and S divide each side of the quadrilateral in half as shown below.



Prove, using vectors, that $PQRS$ is a parallelogram. *(3 marks)*

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23. Vectors, EXT1 V1 SM-Bank 25

Consider the vectors given by $\underset{\sim}{a} = m\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} + m\underset{\sim}{j}$, where $m \in R$.

Find the value(s) of m if the acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is 30°. *(2 marks)*

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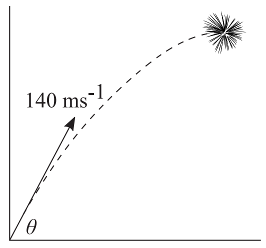
24. Vectors, EXT1 V1 SM-Bank 23

A fireworks rocket is fired from an origin O, with a velocity of 140 metres per second at an angle of θ to the horizontal plane.



The position vector $\underset{\sim}{s}(t)$, from O, of the rocket after t seconds is given by

$$\underset{\sim}{s} = 140t\cos\theta\underset{\sim}{i} + \left(140t\sin\theta - 4.9t^2\right)\underset{\sim}{j}$$

The rocket explodes when it reaches its maximum height.

i. Show the rocket explodes at a height of $1000\sin^2\theta$ metres. *(2 marks)*

ii. Show the rocket explodes at a horizontal distance of $1000\sin 2\theta$ metres from O. *(1 mark)*

iii. For best viewing, the rocket must explode at a horizontal distance of 500 m and 800 m from O, and at least 600 m above the ground.

For what values of θ will this occur. *(3 marks)*

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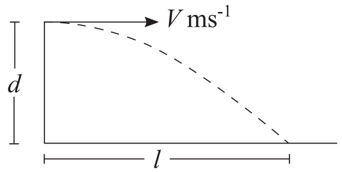
25. Vectors, EXT1 V1 SM-Bank 8

A projectile is fired horizontally off a cliff at an initial speed of V metres per second.



The projectile strikes the water, l metres from the base of the cliff.

Let g be the acceleration due to gravity and assume air resistance is negligible.

i. Show the projectile hits the water when

$t = \sqrt{\dfrac{2d}{g}}$ *(2 marks)*

ii. If l equals twice the height of the cliff, at what angle does the projectile hit the water? *(2 marks)*

iii. Show that the speed at which the projectile hits the water is

$2\sqrt{dg}$ metres per second. *(1 mark)*

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26. Vectors, EXT1 V1 2021 HSC 14a

A plane needs to travel to a destination that is on a bearing of 063°. The engine is set to fly at a constant 175 km/h. However, there is a wind from the south with a constant speed of 42 km/h.

On what constant bearing, to the nearest degree, should the direction of the plane be set in order to reach the destination? *(3 marks)*

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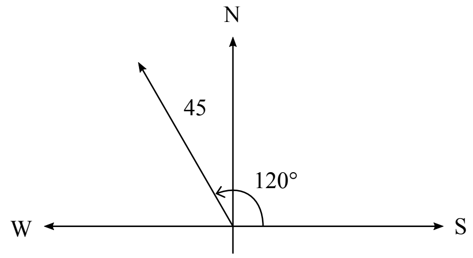
27. Vectors, EXT1 V1 SM-Bank 31

A drone is set to fly west at 38 km/h.

A cross wind diverts its path so that it travels with a speed of 45 km/h in the direction shown below.



Calculate the speed, to one decimal place, and the bearing of the cross wind, to the nearest degree. *(3 marks)*

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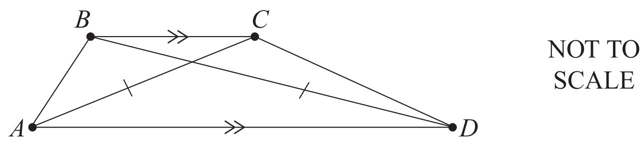
28. Vectors, EXT1 V1 2021 HSC 14c

i. For vector $\underset{\sim}{v}$, show that $\underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|^2$. *(1 mark)*

ii. In the trapezium $ABCD$, BC is parallel to AD and $|\overrightarrow{AC}| = |\overrightarrow{BD}|$.



Let $\underset{\sim}{a} = \overrightarrow{AB}$, $\underset{\sim}{b} = \overrightarrow{BC}$ and $\overrightarrow{AD} = k\overrightarrow{BC}$, where $k > 0$.

Using part (i), or otherwise, show $2\underset{\sim}{a} \cdot \underset{\sim}{b} + (1 - k)|\underset{\sim}{b}|^2 = 0$. *(3 marks)*

Worked Solutions

1. Vectors, EXT1 V1 SM-Bank 16 MC

$\underset{\sim}{a} \perp \underset{\sim}{b} \quad \Rightarrow \quad \underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

$$\begin{aligned} \underset{\sim}{a} \cdot \underset{\sim}{b} &= 2m^2 + m(-1) \\ 0 &= 2m^2 - m \\ 0 &= m(2m - 1) \end{aligned}$$

$\therefore m = 0, \quad m = \dfrac{1}{2}$

$\Rightarrow D$

---

2. Vectors, EXT1 V1 2020 HSC 4 MC

$$\begin{aligned} \underset{\sim}{v} &= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} \\ &= \begin{pmatrix} 9 \\ -2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} |\underset{\sim}{v}| &= \sqrt{9^2 + (-2)^2} \\ &= \sqrt{85} \end{aligned}$$

$\Rightarrow B$

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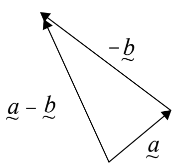
3. Vectors, EXT1 V1 2020 HSC 6 MC

$\underset{\sim}{v} = \underset{\sim}{a} - \underset{\sim}{b}$



$\Rightarrow D$

Worked Solutions

4. Vectors, EXT1 V1 2021 HSC 1 MC

$$\begin{aligned} \overrightarrow{PQ} &= \overrightarrow{OQ} - \overrightarrow{OP} \\ &= \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ 4 \end{pmatrix} \end{aligned}$$

$\Rightarrow C$

---

5. Vectors, EXT1 V1 2021 HSC 5 MC

$\overrightarrow{OA} \cdot \overrightarrow{OB} < 0$

$$\begin{aligned} \left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right| \cos\theta &< 0 \\ \cos\theta &< 0 \end{aligned}$$

If $\cos\theta < 0$, θ is in 2nd quadrant (obtuse).

$\Rightarrow B$

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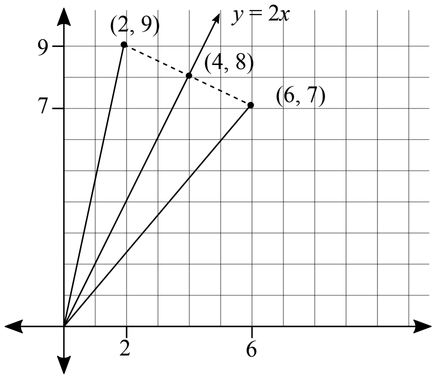
6. Vectors, EXT1 V1 2020 HSC 9 MC

Graph the projection and reflection:



$\Rightarrow B$

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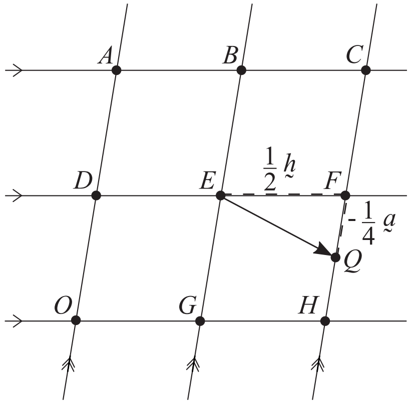
7. Vectors, EXT1 V1 EQ-Bank 4 MC



$\overrightarrow{EQ} = \frac{1}{2}\underset{\sim}{h} - \frac{1}{4}\underset{\sim}{a}$

$\Rightarrow A$

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8. Vectors, EXT1 V1 2020 HSC 11b

$$\begin{aligned} \begin{pmatrix} a \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2a-3 \\ 2 \end{pmatrix} &= 0 \\ a(2a-3) + (-1) \times 2 &= 0 \\ 2a^2 - 3a - 2 &= 0 \\ (2a+1)(a-2) &= 0 \end{aligned}$$

$$\therefore a = -\frac{1}{2} \text{ or } 2$$

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9. Vectors, EXT1 V1 EQ-Bank 4

i. $\underset{\sim}{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}, \ \underset{\sim}{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \ \underset{\sim}{c} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$$(\underset{\sim}{b} + \underset{\sim}{c}) = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

$$\begin{aligned} \underset{\sim}{a} \cdot (\underset{\sim}{b} + \underset{\sim}{c}) &= \begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 7 \end{pmatrix} \\ &= (4 \times 1) - (1 \times 7) \\ &= -3 \end{aligned}$$

ii.
$$\begin{aligned} \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{c} &= \begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 5 \end{pmatrix} \\ &= (4 \times 3) - (1 \times 2) + (4 \times -2) - (1 \times 5) \\ &= -3 \\ &= \underset{\sim}{a} \cdot (\underset{\sim}{b} + \underset{\sim}{c}) \end{aligned}$$

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10. Vectors, EXT1 V1 EQ-Bank 7

$m\underset{\sim}{a} + n\underset{\sim}{b} = \underset{\sim}{c}$

$$m\begin{pmatrix} 6 \\ 2 \end{pmatrix} + n\begin{pmatrix} 1 \\ -5 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$6m + n = 4 \ \ldots (1)$

$2m - 5n = 4 \ \ldots (2)$

Multiply $(2) \times 3$

$6m - 15n = 12 \ \ldots (3)$

Subtract $(1) - (3)$

$16n = -8 \ \Rightarrow \ n = -\dfrac{1}{2}$

Substitute $n = -\dfrac{1}{2}$ into (2):

$2m + \dfrac{5}{2} = 4$

$m = \dfrac{3}{4}$

$\therefore m = \dfrac{3}{4}, \ n = -\dfrac{1}{2}$

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11. Vectors, EXT1 V1 EQ-Bank 27

Since $OABC$ is a rhombus

$|\underset{\sim}{a}| = |\underset{\sim}{c}|$

Diagonals are $\overrightarrow{OB}$ and $\overrightarrow{AC}$, where

$\overrightarrow{OB} = \underset{\sim}{a} + \underset{\sim}{c}$

$\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

$$\begin{aligned}(\underset{\sim}{c} - \underset{\sim}{a}) \cdot (\underset{\sim}{a} + \underset{\sim}{c}) &= \underset{\sim}{c} \cdot \underset{\sim}{a} + \underset{\sim}{c} \cdot \underset{\sim}{c} - \underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{c} \\ &= |\underset{\sim}{c}|^2 - |\underset{\sim}{a}|^2 \\ &= 0\end{aligned}$$

$\therefore \ \overrightarrow{OB} \perp \overrightarrow{AC}$

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12. Vectors, EXT1 V1 SM-Bank 20

i. $\underset{\sim}{a} = \underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$

$$|\underset{\sim}{a}| = \sqrt{1 + (\sqrt{3})^2} = 2$$

$$\hat{a} = \frac{\underset{\sim}{a}}{|\underset{\sim}{a}|} = \frac{1}{2}\left(\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}\right)$$

ii. Solution 1

$\underset{\sim}{a} \Rightarrow$ Position vector from O to $(1, \sqrt{3})$

$\tan\theta = \sqrt{3}$

$\therefore \theta = 60°$

Solution 2

Angle with x-axis = angle with $\underset{\sim}{b} = \underset{\sim}{i}$

$\underset{\sim}{a} \cdot \underset{\sim}{i} = 1 \times 1 = 1$

$\underset{\sim}{a} \cdot \underset{\sim}{i} = |\underset{\sim}{a}||\underset{\sim}{i}|\cos\theta$

$1 = 2 \times 1 \times \cos\theta$

$\cos\theta = \frac{1}{2}$

$\therefore \theta = 60°$

iii. $\underset{\sim}{b} = m\underset{\sim}{i} - 2\underset{\sim}{j}$

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} \cdot \begin{bmatrix} m \\ -2 \end{bmatrix} = m - 2\sqrt{3}$$

Since $\underset{\sim}{a} \perp \underset{\sim}{b}$:

$m - 2\sqrt{3} = 0$

$m = 2\sqrt{3}$

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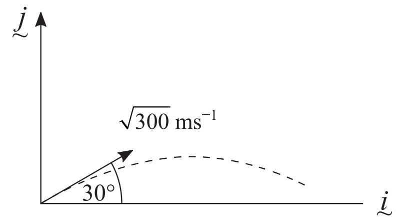
13. Vectors, EXT1 V1 EQ-Bank 28

i.



$$\dot{x} = \sqrt{300}\cos 30^\circ = 10\sqrt{3} \times \frac{\sqrt{3}}{2} = 15 \text{ ms}^{-1}$$

$$x = \int 15\, dt = 15t + C_1$$

When $t = 0,\ x = 0 \Rightarrow C_1 = 0$

$\therefore x = 15t$

ii. $\dot{y} = \int -10\, dt = -10t + C_1$

When $t = 0,\ \dot{y} = 10\sqrt{3}\sin 30^\circ = 5\sqrt{3} \Rightarrow C_1 = 5\sqrt{3}$

$\therefore \dot{y} = 5\sqrt{3} - 10t$

$$y = \int \dot{y}\, dt = 5\sqrt{3}t - 5t^2 + C_2$$

When $t = 0,\ y = 0 \Rightarrow C_2 = 0$

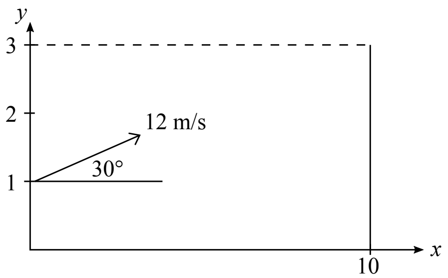
$\therefore y = 5\sqrt{3}t - 5t^2$

iii. $x = 15t \Rightarrow t = \frac{x}{15}$

$$y = 5\sqrt{3} \times \frac{x}{15} - 5\left(\frac{x}{15}\right)^2$$

$$\therefore y = \frac{\sqrt{3}}{3}x - \frac{x^2}{45}$$

14. Vectors, EXT1 V1 2021 HSC 13b



$$r(t) = (Vt\cos\theta)\underset{\sim}{i} + \left(-5t^2 + Vt\sin\theta + h\right)\underset{\sim}{j}$$

$$r'(t) = (V\cos\theta)\underset{\sim}{i} + (-10t + V\sin\theta)\underset{\sim}{j}$$

Max height occurs when $\dot{y} = 0$:

$$10t = V\sin\theta$$

$$t = \frac{12 \times \sin 30^\circ}{10}$$

$$= 0.6 \text{ sec}$$

Find y when $t = 0.6$:

$$y = -5(0.6)^2 + 12 \times 0.6 \times \sin 30^\circ + 1$$

$$= 2.8 \text{ m} < 3 \text{ m}$$

$\therefore$ Ball will not hit ceiling.

Find time of flight when $y = 0$:

$$-5t^2 + 6t + 1 = 0$$

$$5t^2 - 6t - 1 = 0$$

$$t = \frac{6 \pm \sqrt{(-6)^2 + 4 \cdot 5 \cdot 1}}{10}$$

$$= \frac{6 + \sqrt{56}}{10}$$

$$= 1.348\ldots$$

Find x when $t = 1.348$:

$$x = 12 \times 1.348 \times \cos 30$$

$$= 14.0 > 10$$

$\therefore$ Ball will hit the wall on the full.

---

15. Vectors, EXT1 V1 EQ-Bank 2

$$|\underset{\sim}{r}| = |\underset{\sim}{s}| = |\underset{\sim}{t}| \quad \text{(radii)}$$

$$\underset{\sim}{r} = -\underset{\sim}{t}$$

$$\overrightarrow{RS} = \underset{\sim}{s} - \underset{\sim}{r}$$

$$\overrightarrow{TS} = \underset{\sim}{s} - \underset{\sim}{t}$$

$$\overrightarrow{RS} \cdot \overrightarrow{TS} = (\underset{\sim}{s} - \underset{\sim}{r}) \cdot (\underset{\sim}{s} - \underset{\sim}{t})$$

$$= (\underset{\sim}{s} - \underset{\sim}{r}) \cdot (\underset{\sim}{s} + \underset{\sim}{r}) \quad \left(\text{using } \underset{\sim}{r} = -\underset{\sim}{t}\right)$$

$$= \underset{\sim}{s} \cdot (\underset{\sim}{s} + \underset{\sim}{r}) - \underset{\sim}{r} \cdot (\underset{\sim}{s} + \underset{\sim}{r})$$

$$= \underset{\sim}{s} \cdot \underset{\sim}{s} + \underset{\sim}{s} \cdot \underset{\sim}{r} - \underset{\sim}{r} \cdot \underset{\sim}{s} - \underset{\sim}{r} \cdot \underset{\sim}{r}$$

$$= |\underset{\sim}{s}|^2 - |\underset{\sim}{r}|^2$$

$$= 0$$

$\therefore \overrightarrow{RS} \perp \overrightarrow{TS}$

$\therefore \angle RST$ is a right angle.

---

16. Vectors, EXT1 V1 EQ-Bank 3

i. $\underset{\sim}{F} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \ \underset{\sim}{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$$\hat{\underset{\sim}{v}} = \frac{\underset{\sim}{v}}{|\underset{\sim}{v}|} = \frac{\underset{\sim}{v}}{\sqrt{4^2 + 3^2}} = \frac{1}{5}\underset{\sim}{v} = \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$$

$$\begin{aligned} \underset{\sim}{F} \cdot \hat{\underset{\sim}{v}} &= \begin{pmatrix} 3 \\ 6 \end{pmatrix}\begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix} \\ &= 3 \times 0.8 + 6 \times 0.6 \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{proj}_{\underset{\sim}{v}} \underset{\sim}{F} &= (\underset{\sim}{F} \cdot \hat{\underset{\sim}{v}})\hat{\underset{\sim}{v}} \\ &= 6\begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix} \\ &= \begin{pmatrix} 4.8 \\ 3.6 \end{pmatrix} \end{aligned}$$

ii. Component of $\underset{\sim}{F} \perp l$

$$\begin{aligned} &= \begin{pmatrix} 3 \\ 6 \end{pmatrix} - \begin{pmatrix} 4.8 \\ 3.6 \end{pmatrix} \\ &= \begin{pmatrix} -1.8 \\ 2.4 \end{pmatrix} \end{aligned}$$

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17. Vectors, EXT1 V1 EQ-Bank 5

$\underset{\sim}{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \ \underset{\sim}{b} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}, \ \underset{\sim}{c} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \ \underset{\sim}{d} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -9 \end{pmatrix}$$

$$\overrightarrow{CD} = \underset{\sim}{d} - \underset{\sim}{c} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$\begin{aligned} \cos\theta &= \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}| \cdot |\overrightarrow{CD}|} \\ &= \frac{2 + 63}{\sqrt{82} \cdot \sqrt{53}} \\ &= 0.985\ldots \end{aligned}$$

$$\begin{aligned} \therefore \theta &= \cos^{-1} 0.985... \\ &= 9.605\ldots \\ &= 9.6^\circ \ \ \text{(to 1 d.p.)} \end{aligned}$$

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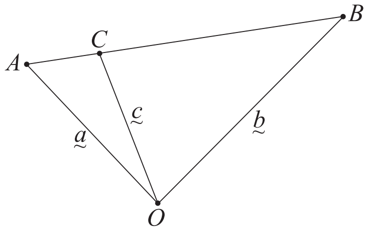
18. Vectors, EXT1 V1 EQ-Bank 6

i.



$$\underset{\sim}{c} = \underset{\sim}{a} + \overrightarrow{AC}$$

$$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$$

$$\overrightarrow{AC} = \lambda\overrightarrow{AB} = \lambda(\underset{\sim}{b} - \underset{\sim}{a})$$

$$\therefore \underset{\sim}{c} = \underset{\sim}{a} + \lambda(\underset{\sim}{b} - \underset{\sim}{a}) = \lambda\underset{\sim}{b} + (1-\lambda)\underset{\sim}{a}$$

ii.

$$\begin{aligned}\overrightarrow{BC} &= \underset{\sim}{c} - \underset{\sim}{b}\\ &= \lambda\underset{\sim}{b} + (1-\lambda)\underset{\sim}{a} - \underset{\sim}{b}\\ &= (1-\lambda)\underset{\sim}{a} + (\lambda - 1)\underset{\sim}{b}\\ &= (1-\lambda)\underset{\sim}{a} - (1-\lambda)\underset{\sim}{b}\\ &= (1-\lambda)(\underset{\sim}{a} - \underset{\sim}{b})\end{aligned}$$

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19. Vectors, EXT1 V1 SM-Bank 13

$$\underset{\sim}{a} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}, \quad \underset{\sim}{b} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$\begin{aligned}\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} &= \frac{28+3}{49+1}(7\underset{\sim}{i} - \underset{\sim}{j})\\ &= \frac{31}{50}(7\underset{\sim}{i} - \underset{\sim}{j})\\ &= \frac{217}{50}\underset{\sim}{i} - \frac{31}{50}\underset{\sim}{j}\end{aligned}$$

$$\begin{aligned}\left|\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}\right| &= \sqrt{\left(\frac{217}{50}\right)^2 + \left(\frac{31}{50}\right)^2}\\ &= \frac{\sqrt{(217^2 + 31^2)}}{50}\\ &= \frac{\sqrt{48\,050}}{50}\\ &= \frac{155\sqrt{2}}{50}\\ &= \frac{31\sqrt{2}}{10}\end{aligned}$$

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20. Vectors, EXT1 V1 SM-Bank 18

$$\underset{\sim}{a} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \underset{\sim}{b} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$|\underset{\sim}{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$|\underset{\sim}{b}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$\cos\theta = \frac{1 \times 3 + 3 \times 1}{\sqrt{10}\sqrt{10}}$$

$$= \frac{3}{5}$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$= 2\left(\frac{3}{5}\right)^2 - 1$$

$$= -\frac{7}{25}$$

---

21. Vectors, EXT1 V1 SM-Bank 21

Angle between $\overrightarrow{BA}$ and $\overrightarrow{BC} = \dfrac{\pi}{3}$

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$

$$= \begin{bmatrix} -a \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -a - 3 \\ 0 \end{bmatrix}$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = \begin{bmatrix} -4 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -a - 3 \\ 0 \end{bmatrix}$$

$$= 4a + 12$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = \left|\overrightarrow{BA}\right| \cdot \left|\overrightarrow{BC}\right| \cos\theta$$

$$4a + 12 = \sqrt{(-4)^2 + (-3)^2} \cdot \sqrt{(-a-3)^2} \cdot \cos\frac{\pi}{3}$$

$$4a + 12 = 5(-a-3) \cdot \frac{1}{2}$$

$$4a + 12 = -\frac{5a}{2} - \frac{15}{2}$$

$$\frac{13a}{2} = -\frac{39}{2}$$

$$\therefore a = -3$$

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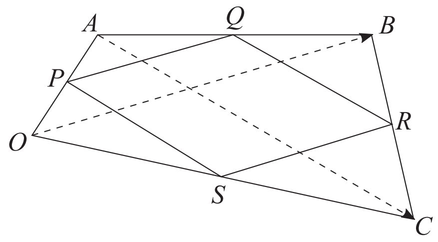
22. Vectors, EXT1 V1 EQ-Bank 26



Consider diagonal $\overrightarrow{OB}$:

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OC} + \overrightarrow{CB}$$

$$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} = \frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{AB}\right) = \frac{1}{2}\overrightarrow{OB}$$

$$\overrightarrow{SR} = \overrightarrow{SC} + \overrightarrow{CR} = \frac{1}{2}\left(\overrightarrow{OC} + \overrightarrow{CB}\right) = \frac{1}{2}\overrightarrow{OB}$$

$$\therefore \overrightarrow{PQ} = \overrightarrow{SR}$$

Consider diagonal $\overrightarrow{AC}$:

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AO} + \overrightarrow{OC}$$

$$\overrightarrow{QR} = \overrightarrow{QB} + \overrightarrow{BR} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \frac{1}{2}\overrightarrow{AC}$$

$$\overrightarrow{PS} = \overrightarrow{PO} + \overrightarrow{OS} = \frac{1}{2}\left(\overrightarrow{AO} + \overrightarrow{OC}\right) = \frac{1}{2}\overrightarrow{AC}$$

$$\therefore \overrightarrow{QR} = \overrightarrow{PS}$$

Since $PQRS$ has equal opposite sides,

$PQRS$ is a parallelogram.

23. Vectors, EXT1 V1 SM-Bank 25

$$\begin{aligned} \underset{\sim}{a} \cdot \underset{\sim}{b} &= m \times 1 + 1 \times m \\ &= 2m \end{aligned}$$

$$\begin{aligned} \underset{\sim}{a} \cdot \underset{\sim}{b} &= |\underset{\sim}{a}||\underset{\sim}{b}|\cos 30^\circ \\ &= \sqrt{m^2 + 1} \cdot \sqrt{1 + m^2} \cdot \cos 30^\circ \\ &= \frac{(m^2 + 1)\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \frac{(m^2 + 1)\sqrt{3}}{2} &= 2m \\ m^2\sqrt{3} + \sqrt{3} &= 4m \\ m^2\sqrt{3} - 4m + \sqrt{3} &= 0 \\ \left(\sqrt{3}m\right)^2 - 4\left(\sqrt{3}m\right) + 3 &= 0 \\ \left(\sqrt{3}m\right)^2 - 4\left(\sqrt{3}m\right) + 2^2 - 1 &= 0 \\ \left(\sqrt{3}m - 2\right)^2 - 1 &= 0 \\ \sqrt{3}m - 2 &= \pm 1 \\ \sqrt{3}m &= 2 \pm 1 \end{aligned}$$

$$\begin{aligned} \therefore m &= \frac{2 \pm 1}{\sqrt{3}} = \frac{3}{\sqrt{3}} \text{ or } \frac{1}{\sqrt{3}} \\ &= \sqrt{3}, \frac{1}{\sqrt{3}} \end{aligned}$$

24. Vectors, EXT1 V1 SM-Bank 23

i. $\underset{\sim}{s} = 140t\cos\theta\underset{\sim}{i} + \left(140t\sin\theta - 4.9t^2\right)\underset{\sim}{j}$

$\underset{\sim}{v} = 140\cos\theta\underset{\sim}{i} + (140\sin\theta - 9.8t)\underset{\sim}{j}$

Max height occurs when $\underset{\sim}{j}$ component of $\underset{\sim}{v} = 0$

$$0 = 140\sin\theta - 9.8t$$

$$t = \frac{140\sin\theta}{9.8}$$

Max height: $\underset{\sim}{j}$ component of $\underset{\sim}{s}$ when $t = \dfrac{140\sin\theta}{9.8}$

$$\begin{aligned}\text{Max height} &= 140\sin\theta \cdot \frac{140\sin\theta}{9.8} - \frac{4.9 \cdot 140^2\sin^2\theta}{9.8^2}\\ &= 2000\sin^2\theta - 1000\sin^2\theta\\ &= 1000\sin^2\theta\end{aligned}$$

ii. Horizontal distance (d):

$\Rightarrow$ $\underset{\sim}{i}$ component of $\underset{\sim}{s}$ when $t = \dfrac{140\sin\theta}{9.8}$

$$\begin{aligned}\therefore d &= 140\cos\theta \cdot \frac{140\sin\theta}{9.8}\\ &= \frac{140 \times 70 \times \sin 2\theta}{9.8}\\ &= 1000\sin 2\theta\end{aligned}$$

iii. Using part ii,

$$500 \leq 1000\sin 2\theta \leq 800$$

$$0.5 \leq \sin 2\theta \leq 0.8$$

In the 1st quadrant:

$$30^\circ \leq 2\theta \leq 53.13^\circ$$

$$15^\circ \leq \theta \leq 26.6^\circ$$

In the 2nd quadrant:

$$126.87^\circ \leq 2\theta \leq 150^\circ$$

$$63.4^\circ \leq \theta \leq 75^\circ$$

When $\theta = 26.6^\circ$:

$$\begin{aligned}\text{Max height} &= 1000 \cdot \sin^2 26.6^\circ\\ &= 200.5 \text{ metres } (< 600 \text{ m})\end{aligned}$$

$\Rightarrow$ Highest max height for $15^\circ \leq \theta < 26.6^\circ$ does not satisfy.

When $\theta = 63.4^\circ$:

$$\begin{aligned}\text{Max height} &= 1000 \cdot \sin^2 63.4^\circ\\ &= 799.5 \text{ metres } (> 600 \text{ m})\end{aligned}$$

$\Rightarrow$ Lowest max height for $63.4^\circ \leq \theta \leq 75^\circ$ satisfies.

$\therefore 63.4^\circ \leq \theta \leq 75^\circ$

25. Vectors, EXT1 V1 SM-Bank 8

i. $\underset{\sim}{v} = V\cos\theta\underset{\sim}{i} + (V\sin\theta - gt)\underset{\sim}{j}$

$= V\cos 0\underset{\sim}{i} + (V\sin 0 - gt)\underset{\sim}{j}$

$= V\underset{\sim}{i} - gt\underset{\sim}{j}$

$$\underset{\sim}{s} = \int \underset{\sim}{v}\, dt$$

$$= Vt\underset{\sim}{i} - \frac{1}{2}gt^2\underset{\sim}{j} + c$$

When $t = 0$, $\underset{\sim}{s} = 0$, $c = 0$

Time of flight:

$\underset{\sim}{j}$ component of $\underset{\sim}{s} = -d$

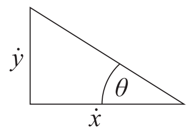
$$-\frac{1}{2}gt^2 = -d$$

$$t^2 = \frac{2d}{g}$$

$$t = \sqrt{\frac{2d}{g}}$$

ii. $l = 2d$ (given)

Projectile hits water at θ:



$\dot{y} = \underset{\sim}{j}$ component of $\underset{\sim}{v}$

$= -gt$

$$= -g \cdot \sqrt{\frac{2d}{g}}$$

$= -\sqrt{2dg}$

$\dot{x} = \underset{\sim}{i}$ component of $\underset{\sim}{v}$

$= V$

When $t = \sqrt{\dfrac{2d}{g}}$,

$\underset{\sim}{i}$ component of $\underset{\sim}{s} = 2d$

$$2d = V \cdot \sqrt{\frac{2d}{g}}$$

$$V = \frac{2d\sqrt{g}}{\sqrt{2d}} = \sqrt{2dg}$$

$$\tan\theta = \frac{|\dot{y}|}{|\dot{x}|} = \frac{\sqrt{2dg}}{\sqrt{2dg}} = 1$$

$\therefore \theta = 45^\circ$

iii. Speed = magnitude of velocity

$$|\underset{\sim}{v}| = \sqrt{\left(\sqrt{2dg}\right)^2 + \left(\sqrt{2dg}\right)^2}$$

$= \sqrt{4dg}$

$= 2\sqrt{dg}$

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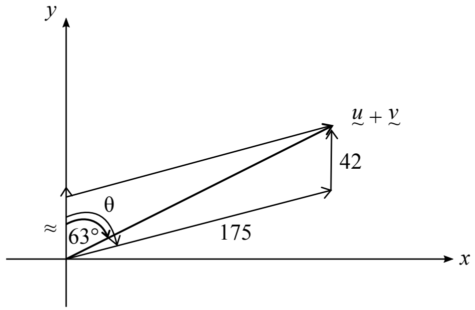
26. Vectors, EXT1 V1 2021 HSC 14a



$\underset{\sim}{u} = (175 \sin \theta, 175 \cos \theta)$

$\underset{\sim}{v} = (0, 42)$

$\underset{\sim}{u} + \underset{\sim}{v} = (175 \sin \theta, 175 \cos \theta + 42)$

$\therefore \tan 63^\circ = \dfrac{175 \sin \theta}{175 \cos \theta + 42}$

$$(175 \cos \theta + 42)\tan 63^\circ = 175 \sin \theta$$

$$175 \cos \theta \tan 63^\circ + 42 \tan 63^\circ = 175 \sin \theta$$

$$175 \sin \theta - 175 \cos \theta \tan 63^\circ = 42 \tan 63^\circ$$

$$\sin \theta - \cos \theta \frac{\sin 63^\circ}{\cos 63^\circ} = \frac{6}{25} \frac{\sin 63^\circ}{\cos 63^\circ}$$

$$\sin \theta \cos 63^\circ - \cos \theta \sin 63^\circ = \frac{6}{25} \sin 63^\circ$$

$$\sin(\theta - 63^\circ) = \frac{6}{25} \sin 63^\circ$$

$$\theta - 63^\circ = \sin^{-1}\left(\frac{6}{25} \sin 63^\circ\right)$$

$$\theta - 63^\circ = 12^\circ$$

$$\therefore \theta \approx 75^\circ$$

♦♦ *Mean mark 31%.*

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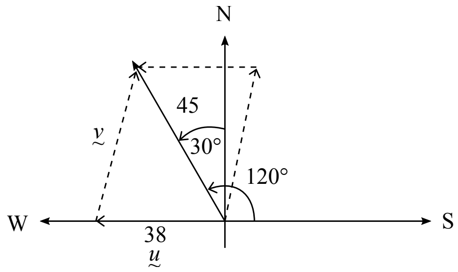
27. Vectors, EXT1 V1 SM-Bank 31



$$\underset{\sim}{u} + \underset{\sim}{v} = (-45 \sin 30,\ 45 \cos 30)$$

$$= \left(-22.5,\ \frac{45\sqrt{3}}{2}\right)$$

$\underset{\sim}{u} = (-38, 0)$, $\underset{\sim}{v} = (x, y)$

$$\begin{pmatrix} -38 \\ 0 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -22.5 \\ \frac{45\sqrt{3}}{2} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15.5 \\ \frac{45\sqrt{3}}{2} \end{pmatrix}$$

$$|\underset{\sim}{v}| = \sqrt{15.5^2 + \frac{45^2 \times 3}{4}}$$

$$= 41.94...$$

$$= 41.9 \text{ km/h}$$

$$\tan \theta = \frac{\frac{45\sqrt{3}}{2}}{15.5} = 2.514$$

$$\theta = 68.3^\circ$$

$\therefore$ The crosswind has a speed of 41.9 km/h on a bearing of 022°.

28. Vectors, EXT1 V1 2021 HSC 14c

i. Let $\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$|\underset{\sim}{v}| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{v} \cdot \underset{\sim}{v} = x^2 + y^2 = \left(\sqrt{x^2 + y^2}\right)^2 = |\underset{\sim}{v}|^2$$

ii. Show $2\underset{\sim}{a} \cdot \underset{\sim}{b} + (1-k)|\underset{\sim}{b}|^2 = 0$

♦♦♦ *Mean mark part (ii) 18%.*

$$\left|\overrightarrow{AC}\right| = \left|\overrightarrow{BD}\right|$$

$$|\underset{\sim}{a} + \underset{\sim}{b}| = |k\underset{\sim}{b} - \underset{\sim}{a}|$$

$$|\underset{\sim}{a} + \underset{\sim}{b}|^2 = |k\underset{\sim}{b} - \underset{\sim}{a}|^2$$

$$(\underset{\sim}{a} + \underset{\sim}{b})(\underset{\sim}{a} + \underset{\sim}{b}) = (k\underset{\sim}{b} - \underset{\sim}{a})(k\underset{\sim}{b} - \underset{\sim}{a}) \quad \text{(see part a.)}$$

$$\underset{\sim}{a} \cdot \underset{\sim}{a} + 2\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{b} = k^2\underset{\sim}{b} \cdot \underset{\sim}{b} - 2k\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a}$$

$$2\underset{\sim}{a} \cdot \underset{\sim}{b} + 2k\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{b} - k^2\underset{\sim}{b} \cdot \underset{\sim}{b} = 0$$

$$2\underset{\sim}{a} \cdot \underset{\sim}{b}(1+k) + \underset{\sim}{b} \cdot \underset{\sim}{b}\left(1 - k^2\right) = 0$$

$$2\underset{\sim}{a} \cdot \underset{\sim}{b}(1+k) + \underset{\sim}{b} \cdot \underset{\sim}{b}(1+k)(1-k) = 0$$

$$2\underset{\sim}{a} \cdot \underset{\sim}{b} + (1-k)|\underset{\sim}{b}|^2 = 0$$