MATHEMATICS EXTENSION 2

SmarterEd Maths & Sciences

COMPREHENSIVE HSC REVISION SERIES 2025

Thank you for subscribing to SmarterMaths Teacher Edition (Silver) in 2025.

Key features of the Extension 2 “2025 HSC Comprehensive Revision Series” include:

- ~17 hours of cherry-picked HSC revision questions by topic.
- Targeted at motivated students aiming for a Band 5 or 6 equivalent result.
- Weighting toward more difficult examples.
- Mark allocations given to each topic generally reflect its historical (new syllabus) HSC exam allocation.
- ***Attempt, carefully review and annotate*** this revision set in Term 3.
- This question set provides the foundation of a concise and high-quality revision resource for the run into the HSC exam.
- This resource should be used to complement (not replace) the critical final stretch preparation for every student – completing full practice papers in exam conditions.

Our analysis on each topic, the common question types, past areas of difficulty and recent HSC trends all combine to create this revision set that ensures students cover a wide cross-section of the key areas.

IMPORTANT: If students have been exposed to questions in these worksheets during the year, we say great. Many top performing students attest to the benefits of doing quality questions 2-3 times before the HSC. This type of revision set is aimed at creating confidence and *speed through the exam*, with cherry picked questions that cover all important elements of revision while avoiding low percentage rabbit hole excursions.

HSC Final Study: V1 Vectors (2 of 2) (~18.0% historical contribution)

Key Areas addressed by this worksheet

Vectors and Geometry (8.4%)

- *Vectors and Geometry* is the 600-pound Silverback in the Ext2 rainforest that attracted multiple high mark, high difficulty questions in both 2024 and 2023.
- These allocations need to be added to impressive contributions each year in the prior period 2020-2022. In short, its revision importance for achieving a band 5-6 result cannot be overstated.
- Revision questions include multiple examples of both 2D and 3D shapes which have both featured in new syllabus exams.
- The most common question type looks at pyramids, both square (2021) and triangular (2023, 2022).
- 2D quadrilaterals and triangles were the subject of *2024 Q14e*, *2023 Q11d* and *2020 Q15b* which deserve close revision attention.
- *2023 Q15c* and *2021 Q16a* look at spheres and both caused major issues. Important revision examples.

> *“When the mind grows weary from vectors and geometry, step outside and look up and let clarity come not by force, but by space.”*
>
> *~ Marcus Aurelius*

EXTENSION 2 2025

HSC Revision Series

Smarter Ed

Vectors

V1 Working With Vectors (2 of 2)

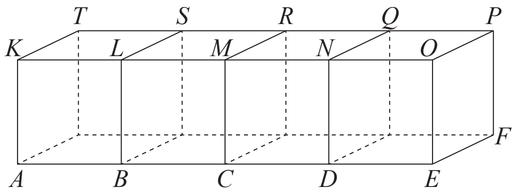
Vectors and Geometry

Exam Equivalent Time: 90 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Vectors, EXT2 V1 2021 HSC 1 MC

Four cubes are placed in a line as shown on the diagram.



Which of the following vectors is equal to $\overrightarrow{AB} + \overrightarrow{CQ}$

A. $\overrightarrow{AQ}$

B. $\overrightarrow{CP}$

C. $\overrightarrow{PB}$

D. $\overrightarrow{RA}$

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2. Vectors, EXT2 V1 2023 HSC 10 MC

Consider any three-dimensional vectors $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\underset{\sim}{c} = \overrightarrow{OC}$ that satisfy these three conditions

$\underset{\sim}{a} \cdot \underset{\sim}{b} = 1$

$\underset{\sim}{b} \cdot \underset{\sim}{c} = 2$

$\underset{\sim}{c} \cdot \underset{\sim}{a} = 3.$

Which of the following statements about the vectors is true?

A. Two of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ could be unit vectors.

B. The points A, B and C could lie on a sphere centred at O.

C. For any three-dimensional vector $\underset{\sim}{a}$, vectors $\underset{\sim}{b}$ and $\underset{\sim}{c}$ can be found so that $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ satisfy these three conditions.

D. $\forall$ $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ satisfying the conditions, $\exists$ r, s and t such that r, s and t are positive real numbers and $r\underset{\sim}{a} + s\underset{\sim}{b} + t\underset{\sim}{c} = \underset{\sim}{0}$.

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3. Vectors, EXT2 V1 EQ-Bank 9

A parallelogram is formed by joining the points $P(-2,1,4)$, $Q(1,4,5)$, $R(0,2,3)$ and $S(a,b,c)$.

Use vector methods to find a, b and c. *(2 marks)*

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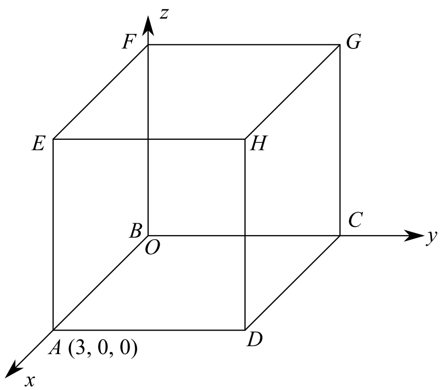
4. Vectors, EXT2 V1 SM-Bank 23

A cube with side length 3 units is pictured below.



i. Calculate the magnitude of vector $\overrightarrow{AG}$. ***(1 mark)***

ii. Find the acute angle between the diagonals $\overrightarrow{AG}$ and $\overrightarrow{BH}$. ***(3 marks)***

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5. Vectors, EXT2 V1 2019 SPEC1-N 5

A triangle has vertices $A\left(\sqrt{3}+1, -2, 4\right)$, $B(1, -2, 3)$ and $C\left(2, -2, \sqrt{3}+3\right)$.

i. Find angle ABC ***(3 marks)***

ii. Find the area of the triangle. ***(2 marks)***

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6. Vectors, EXT2 V1 2023 HSC 11d

The quadrilaterals $ABCD$ and $ABEF$ are parallelograms.

By considering $\overrightarrow{AB}$, show that $CDFE$ is also a parallelogram. ***(2 marks)***

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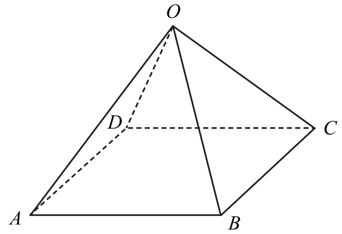
7. Vectors, EXT2, V1 SM-Bank 16



Let $OABCD$ be a right square pyramid where $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$, $\underset{\sim}{c} = \overrightarrow{OC}$ and $\underset{\sim}{d} = \overrightarrow{OD}$.

Show that $\underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{b} + \underset{\sim}{d}$. ***(3 marks)***

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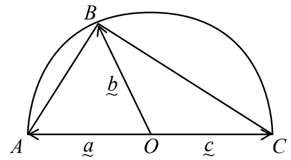
8. Vectors, EXT2 V1 EQ-Bank 15

Point B sits on the arc of a semi-circle with diameter AC.



Using vectors, show $\angle ABC$ is a right angle. ***(2 marks)***

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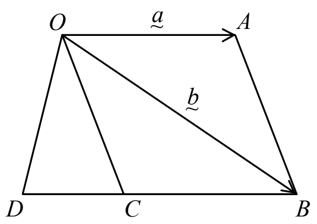
9. Vectors, EXT2 V1 SM-Bank 20

$OABD$ is a trapezium in which $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

OC is parallel to AB and $DC:CB = 1:2$



Using vectors, express $\overrightarrow{DA}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. *(3 marks)*

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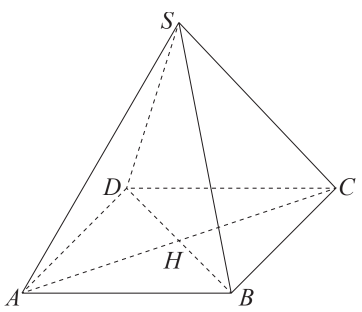
10. Vectors, EXT2 V1 2021 HSC 12e

The diagram shows the pyramid $ABCDS$ where $ABCD$ is a square. The diagonals of the square bisect each other at H.



i. Show that $\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} = \underset{\sim}{0}$ *(1 mark)*

Let G be the point such that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} + \overrightarrow{GS} = \underset{\sim}{0}$.

ii. Using part (i), or otherwise, show that $4\overrightarrow{GH} + \overrightarrow{GS} = \underset{\sim}{0}$. *(2 marks)*

iii. Find the value of λ such that $\overrightarrow{HG} = \lambda\overrightarrow{HS}$ *(1 mark)*

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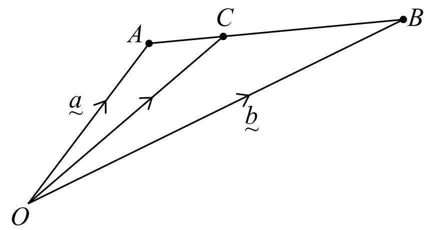
11. Vectors, EXT2 V1 2020 HSC 15b

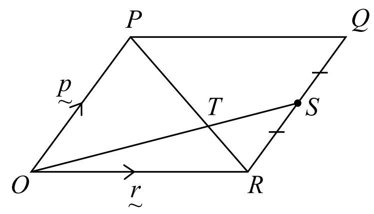
The point C divides the interval AB so that $\dfrac{CB}{AC} = \dfrac{m}{n}$. The position vectors of A and B are $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively, as shown in the diagram.



i. Show that $\overrightarrow{AC} = \dfrac{n}{m+n}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$. *(2 marks)*

ii. Prove that $\overrightarrow{OC} = \dfrac{m}{m+n}\underset{\sim}{a} + \dfrac{n}{m+n}\underset{\sim}{b}$. *(1 mark)*

Let $OPQR$ be a parallelogram with $\overrightarrow{OP} = \underset{\sim}{p}$ and $\overrightarrow{OR} = \underset{\sim}{r}$. The point S is the midpoint of QR and T is the intersection of PR and OS, as shown in the diagram.



iii. Show that $\overrightarrow{OT} = \dfrac{2}{3}\underset{\sim}{r} + \dfrac{1}{3}\underset{\sim}{p}$. *(3 marks)*

iv. Using parts (ii) and (iii), or otherwise, prove that T is the point that divides the interval PR in the ratio $2:1$. *(1 mark)*

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12. Vectors, EXT2 V1 2024 HSC 14e

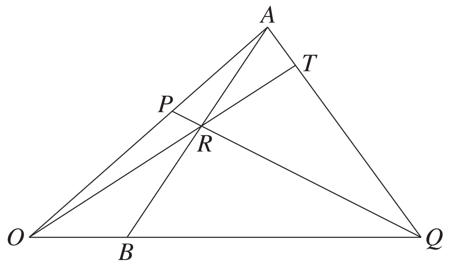
The diagram shows triangle OQA.

The point P lies on OA so that $OP:OA = 3:5$.

The point B lies on OQ so that $OB:OQ = 1:3$.

The point R is the intersection of AB and PQ.

The point T is chosen on AQ so that O, R and T are collinear.



Let $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\overrightarrow{PR} = k\overrightarrow{PQ}$ where k is a real number.

i. Show that $\overrightarrow{OR} = \dfrac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}$. *(2 marks)*

Writing $\overrightarrow{AR} = h\overrightarrow{AB}$, where h is a real number, it can be shown that $\overrightarrow{OR} = (1-h)\underset{\sim}{a} + h\underset{\sim}{b}$. (Do NOT prove this.)

ii. Show that $k = \dfrac{1}{6}$. *(2 marks)*

iii. Find $\overrightarrow{OT}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. *(2 marks)*

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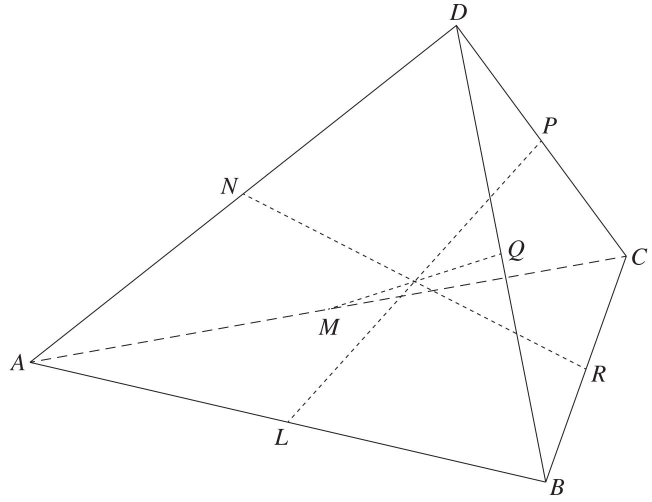
13. Vectors, EXT2 V1 2023 HSC 15b

On the triangular pyramid $ABCD$, L is the midpoint of AB, M is the midpoint of AC, N is the midpoint of AD, P is the midpoint of CD, Q is the midpoint of BD and R is the midpoint of BC.



Let $\underset{\sim}{b} = \overrightarrow{AB}$, $\underset{\sim}{c} = \overrightarrow{AC}$ and $\underset{\sim}{d} = \overrightarrow{AD}$.

i. Show that $\overrightarrow{LP} = \dfrac{1}{2}(-\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d})$. *(1 mark)*

ii. It can be shown that

$$\overrightarrow{MQ} = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{c} + \underset{\sim}{d}) \text{ and}$$

$$\overrightarrow{NR} = \frac{1}{2}(\underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{d}). \text{ (Do NOT prove these.)}$$

Prove that

$$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{BD}\right|^2 + \left|\overrightarrow{CD}\right|^2$$
$$= 4\left(\left|\overrightarrow{LP}\right|^2 + \left|\overrightarrow{MQ}\right|^2 + \left|\overrightarrow{NR}\right|^2\right)$$ *(3 marks)*

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14. Vectors, EXT2 V1 2021 HSC 16a

i. The point $P(x, y, z)$ lies on the sphere of radius 1 centred at the origin O.

Using the position vector of P, $\overrightarrow{OP} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$, and the triangle inequality, or otherwise, show that $|x| + |y| + |z| \geq 1$. *(2 marks)*

ii. Given the vectors $\underset{\sim}{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, show that

$$|a_1b_1 + a_2b_2 + a_3b_3| \leq \sqrt{a_1^2 + a_2^2 + a_3^2}\,\sqrt{b_1^2 + b_2^2 + b_3^2}.$$ *(3 marks)*

iii. As in part (i), the point $P(x, y, z)$ lies on the sphere of radius 1 centred at the origin O.

Using part (ii), or otherwise, show that $|x| + |y| + |z| \leq \sqrt{3}$. *(2 marks)*

15. Vectors, EXT2 V1 2022 HSC 14a

i. The two non-parallel vectors $\vec{u}$ and $\vec{v}$ satisfy $\lambda\vec{u} + \mu\vec{v} = \vec{0}$ for some real numbers λ and μ.

Show that $\lambda = \mu = 0$. *(2 marks)*

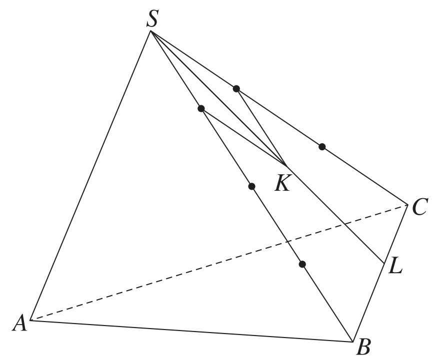
ii. The two non-parallel vectors $\vec{u}$ and $\vec{v}$ satisfy $\lambda_1\vec{u} + \mu_1\vec{v} = \lambda_2\vec{u} + \mu_2\vec{v}$ for some real numbers $\lambda_1, \lambda_2, \mu_1$ and μ_2.

Using part (i), or otherwise, show that $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$. *(1 mark)*

The diagram below shows the tetrahedron with vertices A, B, C and S.

The point K is defined by $\overrightarrow{SK} = \frac{1}{4}\overrightarrow{SB} + \frac{1}{3}\overrightarrow{SC}$, as shown in the diagram.

The point L is the point of intersection of the straight lines SK and BC.



iii. Using part (ii), or otherwise, determine the position of L by showing that $\overrightarrow{BL} = \frac{4}{7}\overrightarrow{BC}$. *(2 marks)*

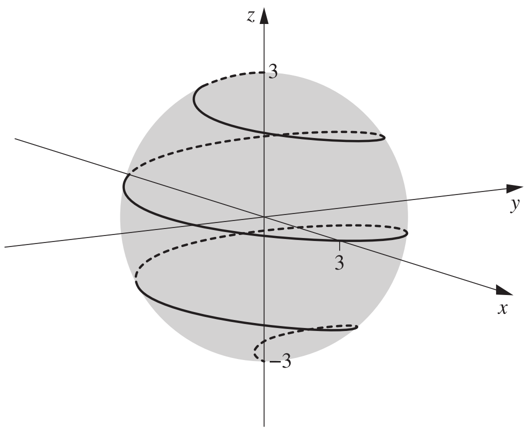
iv. The point P is defined by $\overrightarrow{AP} = -6\overrightarrow{AB} - 8\overrightarrow{AC}$.

Does P lie on the line AL? Justify your answer. *(2 marks)*

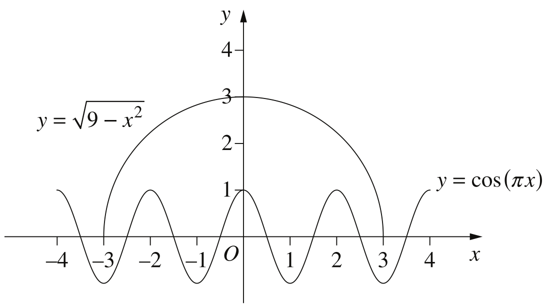
16. Vectors, EXT2 V1 2023 HSC 15c

A curve $\mathcal{C}$ spirals 3 times around the sphere centred at the origin and with radius 3, as shown.

A particle is initially at the point $(0,0,-3)$ and moves along the curve $\mathcal{C}$ on the surface of the sphere, ending at the point $(0,0,3)$.



By using the diagram below, which shows the graphs of the functions $f(x)=\cos(\pi x)$ and $g(x)=\sqrt{9-x^2}$, and considering the graph $y=f(x)g(x)$, give a possible set of parametric equations that describe the curve $\mathcal{C}$. *(3 marks)*



Worked Solutions

1. Vectors, EXT2 V1 2021 HSC 1 MC

$$\begin{aligned}\overrightarrow{AB}+\overrightarrow{CQ}&=\overrightarrow{CD}+\overrightarrow{DP}\\&=\overrightarrow{CP}\end{aligned}$$

$\Rightarrow\ B$

2. Vectors, EXT2 V1 2023 HSC 10 MC

♦♦♦ Mean mark 15%.

By elimination

Option A:

If $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are the two unit vectors, $\underset{\sim}{a}\cdot\underset{\sim}{b}=|\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta=\cos\theta$

$-1\le\cos\theta\le1\ \ \Rightarrow\ \ -1\le\underset{\sim}{a}\cdot\underset{\sim}{b}\le1$

Given $\underset{\sim}{a}\cdot\underset{\sim}{b}=1\ \Rightarrow\ \underset{\sim}{a}=\underset{\sim}{b}\ \Rightarrow\ \underset{\sim}{b}\cdot\underset{\sim}{c}=\underset{\sim}{c}\cdot\underset{\sim}{a}$

Contradicts $\underset{\sim}{b}\cdot\underset{\sim}{c}=2$ and $\underset{\sim}{c}\cdot\underset{\sim}{a}=3$

Similar reasoning rules out any pair satisfying all conditions (eliminate A).

Option C: If $\underset{\sim}{a}=\underset{\sim}{0}$, $\underset{\sim}{a}\cdot\underset{\sim}{b}=0\neq1$ (eliminate C).

Option D:

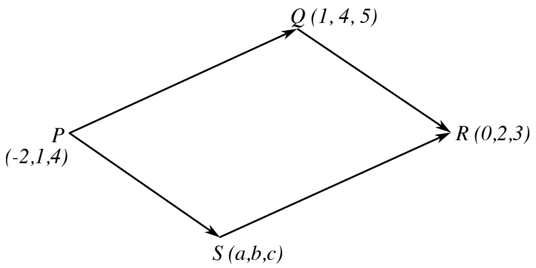
Consider the vectors below that satisfy the conditions,

$$\underset{\sim}{a}=\begin{pmatrix}1\\1\\1\end{pmatrix},\ \underset{\sim}{b}=\begin{pmatrix}1\\0\\0\end{pmatrix},\ \underset{\sim}{c}=\begin{pmatrix}2\\0\\1\end{pmatrix}$$

However, $r\underset{\sim}{a}+s\underset{\sim}{b}+t\underset{\sim}{c}=\underset{\sim}{0}$ requires $r=s=t=0$ which are not positive constants (eliminate D).

$\Rightarrow B$

3. Vectors, EXT2 V1 EQ-Bank 9



Opposite sides of a parallelogram are equal and parallel.

$\Rightarrow \overrightarrow{PQ} = \overrightarrow{SR}$

$$\overrightarrow{PQ} = \begin{pmatrix} 1+2 \\ 4-1 \\ 5-4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$$

$$\overrightarrow{SR} = \begin{pmatrix} -a \\ 2-b \\ 3-c \end{pmatrix}$$

Equating coordinates:

$-a = 3 \quad \Rightarrow \quad a = -3$

$2 - b = 3 \quad \Rightarrow \quad b = -1$

$3 - c = 1 \quad \Rightarrow \quad c = 2$

4. Vectors, EXT2 V1 SM-Bank 23

i. $A(3,0,0), \quad G(0,3,3)$

$$\overrightarrow{AG} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$$

$$\left|\overrightarrow{AG}\right| = \sqrt{9+9+9}$$

$$= 3\sqrt{3} \text{ units}$$

ii. $H(3,3,3)$

$$\overrightarrow{BH} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

$$\overrightarrow{AG} \cdot \overrightarrow{BH} = \left|\overrightarrow{AG}\right| \cdot \left|\overrightarrow{BH}\right| \cos\theta$$

$$\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = \sqrt{9+9+9} \cdot \sqrt{9+9+9}\cos\theta$$

$$-9 + 9 + 9 = 27\cos\theta$$

$$\cos\theta = \frac{1}{3}$$

$$\theta = 70.52\ldots$$

$$= 70^\circ 32'$$

5. Vectors, EXT2 V1 2019 SPEC1-N 5

i. $\overrightarrow{BA} = \begin{pmatrix} \sqrt{3}+1 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ 0 \\ 1 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -2 \\ \sqrt{3}+3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \sqrt{3} \end{pmatrix}$

$$\cos\angle ABC = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|}$$

$$= \frac{2\sqrt{3}}{\sqrt{4}\sqrt{4}}$$

$$= \frac{\sqrt{3}}{2}$$

$$\therefore\ \angle ABC = \frac{\pi}{6}$$

ii. $\text{Area} = \frac{1}{2}\cdot\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|\sin\angle ABC$

$$= \frac{1}{2}\times 2\times 2\times\sin\frac{\pi}{6}$$

$$= 1\ \text{u}^2$$

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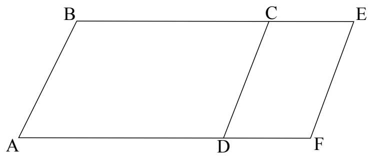
6. Vectors, EXT2 V1 2023 HSC 11d

Parrallelogram ⇒ Show opposite sides are equal

$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{CD}\right|$ ($ABCD$ is a parallelogram)

$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{EF}\right|$ ($ABEF$ is a parallelogram)

$\Rightarrow \left|\overrightarrow{CD}\right| = \left|\overrightarrow{EF}\right|$



$\left|\overrightarrow{CE}\right| = \left|\overrightarrow{BE}\right| - \left|\overrightarrow{BC}\right|$

Similarly,

$\left|\overrightarrow{DF}\right| = \left|\overrightarrow{AF}\right| - \left|\overrightarrow{AD}\right|$

Since $\left|\overrightarrow{BE}\right| = \left|\overrightarrow{AF}\right|$ and $\left|\overrightarrow{BC}\right| = \left|\overrightarrow{AD}\right|$

($ABCD$ and $ABEF$ are parallelograms)

$\Rightarrow \left|\overrightarrow{DF}\right| = \left|\overrightarrow{CE}\right|$

$\therefore CDFE$ is a parallelogram

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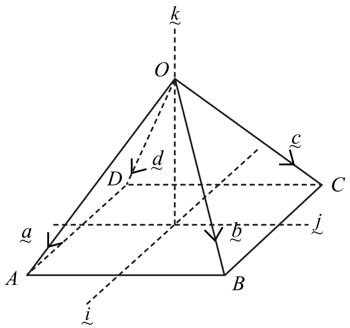
7. Vectors, EXT2, V1 SM-Bank 16



Let $A = (p, -p, -k)$,

$\underset{\sim}{a} = \overrightarrow{OA} = p\underset{\sim}{i} - p\underset{\sim}{j} - q\underset{\sim}{k}$

$\underset{\sim}{b} = \overrightarrow{OB} = p\underset{\sim}{i} + p\underset{\sim}{j} - q\underset{\sim}{k}$

$\underset{\sim}{c} = \overrightarrow{OC} = -p\underset{\sim}{i} + p\underset{\sim}{j} - q\underset{\sim}{k}$

$\underset{\sim}{d} = \overrightarrow{OA} = -p\underset{\sim}{i} - p\underset{\sim}{j} - q\underset{\sim}{k}$

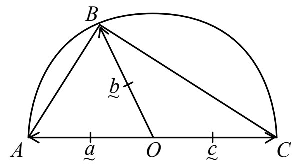
$\underset{\sim}{a} + \underset{\sim}{c} = -2q\underset{\sim}{k}$

$\underset{\sim}{b} + \underset{\sim}{d} = -2q\underset{\sim}{k}$

$\therefore \underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{b} + \underset{\sim}{d}$

8. Vectors, EXT2 V1 EQ-Bank 15

Let $\overrightarrow{OA} = \underset{\sim}{a}$, and $\overrightarrow{OC} = \underset{\sim}{c}$



Prove $\overrightarrow{AB} \perp \overrightarrow{BC}$

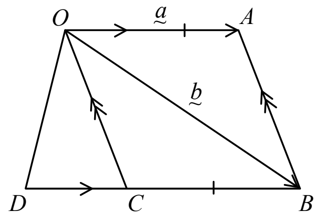
$\left|\underset{\sim}{a}\right| = \left|\underset{\sim}{b}\right| = \left|\underset{\sim}{c}\right|$ (radii)

$\underset{\sim}{c} = -\underset{\sim}{a}$

$$\begin{aligned}
\overrightarrow{AB} \cdot \overrightarrow{BC} &= \left(\underset{\sim}{b} - \underset{\sim}{a}\right)\left(\underset{\sim}{c} - \underset{\sim}{b}\right) \\
&= \underset{\sim}{b} \cdot \underset{\sim}{c} - \left|\underset{\sim}{b}\right|^2 - \underset{\sim}{a} \cdot \underset{\sim}{c} + \underset{\sim}{a} \cdot \underset{\sim}{b} \\
&= \underset{\sim}{b} \cdot \underset{\sim}{c} - \left|\underset{\sim}{b}\right|^2 + \underset{\sim}{c} \cdot \underset{\sim}{c} - \underset{\sim}{c} \cdot \underset{\sim}{b} \\
&= \left|\underset{\sim}{c}\right|^2 - \left|\underset{\sim}{b}\right|^2 \\
&= 0
\end{aligned}$$

$\therefore \angle ABC$ is a right angle.

9. Vectors, EXT2 V1 SM-Bank 20



$\overrightarrow{DA} = \overrightarrow{DB} + \overrightarrow{BA}$

$\overrightarrow{BA} = \underset{\sim}{a} - \underset{\sim}{b}$

Since $OC \,||\, AB$ and $OA \,||\, CB$

$\Rightarrow OABC$ is a parallelogram

$\overrightarrow{OA} = \overrightarrow{CB} = \underset{\sim}{a}$

$\overrightarrow{DC} = \dfrac{1}{2}\underset{\sim}{a}$

$$\begin{aligned}\overrightarrow{DB} &= \overrightarrow{DC} + \overrightarrow{CB} \\ &= \frac{3}{2}\underset{\sim}{a}\end{aligned}$$

$$\begin{aligned}\therefore\ \overrightarrow{DA} &= \frac{3}{2}\underset{\sim}{a} + \left(\underset{\sim}{a} - \underset{\sim}{b}\right) \\ &= \frac{5}{2}\underset{\sim}{a} - \underset{\sim}{b}\end{aligned}$$

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10. Vectors, EXT2 V1 2021 HSC 12e

i. Since diagonal $\overrightarrow{AC}$ is bisected by H:

$\overrightarrow{HA} = -\ \overrightarrow{HC}$

Similarly for diagonal $\overrightarrow{BD}$

$\overrightarrow{HB} = -\overrightarrow{HD}$

$$\begin{aligned}\therefore\ \overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} &= -\overrightarrow{HC} - \overrightarrow{HD} + \overrightarrow{HC} + \overrightarrow{HD} \\ &= \underset{\sim}{0}\end{aligned}$$

ii. $\overrightarrow{GA} = \overrightarrow{GH} + \overrightarrow{HA}\ ,\ \overrightarrow{GB} = \overrightarrow{GH} + \overrightarrow{HB}$
$\overrightarrow{GC} = \overrightarrow{GH} + \overrightarrow{HC}\ ,\ \overrightarrow{GD} = \overrightarrow{GH} + \overrightarrow{HD}$

$$\begin{aligned}&\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} + \overrightarrow{GS} \\ &\quad= \overrightarrow{GH} + \overrightarrow{HA} + \overrightarrow{GH} + \overrightarrow{HB} + \overrightarrow{GH} + \overrightarrow{HC} + \overrightarrow{GH} + \overrightarrow{HD} + \overrightarrow{GS} \\ &\quad= 4\overrightarrow{GH} + \left(\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} + \overrightarrow{GS}\right) \\ &\quad= 4\overrightarrow{GH} + \overrightarrow{GS}\end{aligned}$$

$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} + \overrightarrow{GS} = \underset{\sim}{0}$ (given)

$\therefore 4\overrightarrow{GH} + \overrightarrow{GS} = \underset{\sim}{0}$

iii. $\overrightarrow{HS} = \overrightarrow{HG} + \overrightarrow{GS}$
$\overrightarrow{GS} = \overrightarrow{HS} + \overrightarrow{GH}$

♦ *Mean mark 48%.*

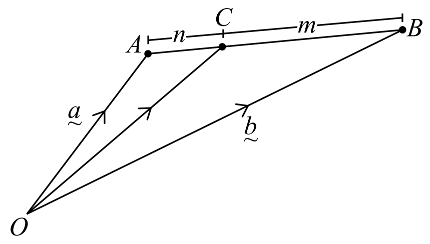
Using part (ii):

$$\begin{aligned}4\overrightarrow{GH} + \overrightarrow{HS} + \overrightarrow{GH} &= \underset{\sim}{0} \\ 5\overrightarrow{GH} &= \overrightarrow{SH} \\ \overrightarrow{GH} &= \frac{1}{5}\overrightarrow{SH} \\ \overrightarrow{HG} &= \frac{1}{5}\overrightarrow{HS}\end{aligned}$$

$\therefore\ \lambda = \dfrac{1}{5}$

11. Vectors, EXT2 V1 2020 HSC 15b

i.



$$\frac{\overrightarrow{AC}}{\overrightarrow{AB}} = \frac{n}{m+n}$$

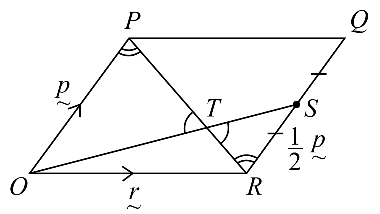
$$\overrightarrow{AC} = \frac{n}{m+n} \cdot \overrightarrow{AB}$$
$$= \frac{n}{m+n}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$$

ii. $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$$= \underset{\sim}{a} + \frac{n}{m+n}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$$
$$= \underset{\sim}{a} - \frac{n}{m+n}\underset{\sim}{a} + \frac{n}{m+n}\underset{\sim}{b}$$
$$= \left(1 - \frac{n}{m+n}\right)\underset{\sim}{a} + \frac{n}{m+n}\underset{\sim}{b}$$
$$= \left(\frac{m+n-n}{m+n}\right)\underset{\sim}{a} + \frac{n}{m+n}\underset{\sim}{b}$$
$$= \frac{m}{m+n}\underset{\sim}{a} + \frac{n}{m+n}\underset{\sim}{b}$$

iii. Show $\overrightarrow{OT} = \frac{2}{3}\underset{\sim}{r} + \frac{1}{3}\underset{\sim}{p}$

♦ *Mean mark part (iii) 50%.*



Consider ΔPTO and ΔRTS:

$\angle PTO = \angle RTS$ (vertically opposite)

$\angle OPT = \angle SRT$ (vertically opposite)

$\therefore\ \Delta PTO\ |||\ \Delta RTS$ (equiangular)

$OT:TS = OP:SR = 2:1$

(corresponding sides in the same ratio)

$$\frac{\overrightarrow{OT}}{\overrightarrow{OS}} = \frac{2}{3}$$

$$\overrightarrow{OT} = \frac{2}{3}\overrightarrow{OS}$$
$$= \frac{2}{3}\left(\underset{\sim}{r} + \frac{1}{2}\underset{\sim}{p}\right)$$
$$= \frac{2}{3}\underset{\sim}{r} + \frac{1}{3}\underset{\sim}{p}$$

Mean mark part (iv) 51%.

iv. Let $\overrightarrow{OT}$ divide PR so that $\dfrac{TR}{PT} = \dfrac{m}{n}$

Using part (ii):

$$\overrightarrow{OT} = \frac{m}{m+n}\underset{\sim}{p} + \frac{n}{m+n}c$$

$$\overrightarrow{OT} = \frac{1}{3}\underset{\sim}{p} + \frac{2}{3}\underset{\sim}{r} \quad \text{(part (iii))}$$

$$\frac{m}{m+n} = \frac{1}{3},\ \frac{n}{m+n} = \frac{2}{3}$$

$\Rightarrow\ m = 1\ ,\ n = 2$

$\therefore\ T$ divides PR in ratio $2:1$.

12. Vectors, EXT2 V1 2024 HSC 14e

i. $\underset{\sim}{a} = \overrightarrow{OA},\ \underset{\sim}{b} = \overrightarrow{OB},\ \overrightarrow{PR} = k\overrightarrow{PQ}$

$\overrightarrow{OP} : \overrightarrow{OA} = 3:5\ \Rightarrow\ \overrightarrow{OP} = \dfrac{3}{5} \times \overrightarrow{OA}$

$\overrightarrow{OB} : \overrightarrow{OQ} = 1:3\ \Rightarrow\ \overrightarrow{OQ} = 3 \times \overrightarrow{OB}$

Show $\overrightarrow{OR} = \dfrac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}$

$$\begin{aligned}\overrightarrow{OR} &= \overrightarrow{OP} + k\overrightarrow{PQ}\\ &= \overrightarrow{OP} + k(\overrightarrow{OQ} - \overrightarrow{OP})\\ &= (1-k)\overrightarrow{OP} + k\overrightarrow{OQ}\\ &= \frac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}\end{aligned}$$

ii. $\overrightarrow{AR} = h\overrightarrow{AB},\quad h \in \mathbb{R}$

$\overrightarrow{OR} = (1-h)\underset{\sim}{a} + h\underset{\sim}{b}$ (given)

$\overrightarrow{OR} = \dfrac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}$ (part(i))

Vector $\underset{\sim}{a} \neq \lambda\underset{\sim}{b}\ (\lambda \in \mathbb{R})\ \Rightarrow$ linearly independent and are basis vectors for $\overrightarrow{OR}$.

Equating coefficients:

$\dfrac{3}{5}(1-k) = 1 - h \ldots (1)$

$h = 3k \ldots (2)$

Substituting (2) into (1)

$$\begin{aligned}\frac{3}{5}(1-k) &= 1 - 3k\\ 3 - 3k &= 5 - 15k\\ k &= \frac{1}{6}\end{aligned}$$

iii. $\overrightarrow{OT} = \lambda\overrightarrow{OR}$

♦ Mean mark (iii) 45%.

Using parts (i) and (ii):

$\overrightarrow{OR} = \dfrac{3}{5}\left(1 - \dfrac{1}{6}\right)\underset{\sim}{a} + 3\left(\dfrac{1}{6}\right)\underset{\sim}{b} = \dfrac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)$

$\overrightarrow{OT} = \dfrac{\lambda}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

Find λ:

$$\begin{aligned}\overrightarrow{OT} &= \overrightarrow{OA} + \mu\overrightarrow{AQ}\\ \frac{\lambda}{2}(\underset{\sim}{a} + \underset{\sim}{b}) &= \underset{\sim}{a} + \mu(3\underset{\sim}{b} - \underset{\sim}{a}) \quad \left(\text{noting } \overrightarrow{AQ} = \overrightarrow{OQ} - \overrightarrow{OA} = 3\underset{\sim}{b} - \underset{\sim}{a}\right)\\ &= \underset{\sim}{a}(1-\mu) + 3\mu\underset{\sim}{b}\end{aligned}$$

Equating coefficients:

$\dfrac{\lambda}{2} = 3\mu\ \Rightarrow\ \mu = \dfrac{\lambda}{6}$

$$\begin{aligned}1 - \frac{\lambda}{6} &= \frac{\lambda}{2}\\ 6 - \lambda &= 3\lambda\\ \lambda &= \frac{3}{2}\end{aligned}$$

$\therefore\ \overrightarrow{OT} = \dfrac{3}{4}(\underset{\sim}{a} + \underset{\sim}{b})$

13. Vectors, EXT2 V1 2023 HSC 15b

i. $\overrightarrow{LP} = \overrightarrow{LA} + \overrightarrow{AC} + \overrightarrow{CP}$

$= \frac{1}{2}\overrightarrow{BA} + \overrightarrow{AC} + \frac{1}{2}\overrightarrow{CD}$

$= -\frac{1}{2}\underset{\sim}{b} + \underset{\sim}{c} + \frac{1}{2}(\underset{\sim}{d} - \underset{\sim}{c})$

$= -\frac{1}{2}\underset{\sim}{b} + \underset{\sim}{c} + \frac{1}{2}\underset{\sim}{d} - \frac{1}{2}\underset{\sim}{c}$

$= \frac{1}{2}(-\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d})$

ii. $\overrightarrow{MQ} = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{c} + \underset{\sim}{d})$

♦♦ *Mean mark (ii) 34%.*

$\overrightarrow{NR} = \frac{1}{2}(\underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{d})$.

$\text{RHS} = 4\left(\left|\overrightarrow{LP}\right|^2 + \left|\overrightarrow{MQ}\right|^2 + \left|\overrightarrow{NR}\right|^2\right)$

$= 4\left(\overrightarrow{LP}\cdot\overrightarrow{LP} + \overrightarrow{MQ}\cdot\overrightarrow{MQ} + \overrightarrow{NR}\cdot\overrightarrow{NR}\right)$

$= 4\Bigg(\frac{1}{4}(-\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d})\cdot(-\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d}) +$

$\frac{1}{4}(\underset{\sim}{b} - \underset{\sim}{c} + \underset{\sim}{d})\cdot(\underset{\sim}{b} - \underset{\sim}{c} + \underset{\sim}{d}) +$

$\frac{1}{4}(\underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{d})\cdot(\underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{d})\Bigg)$

$= (\underset{\sim}{c} + (\underset{\sim}{d} - \underset{\sim}{b}))\cdot(\underset{\sim}{c} + (\underset{\sim}{d} - \underset{\sim}{b})) + (\underset{\sim}{d} + (\underset{\sim}{b} - \underset{\sim}{c}))\cdot(\underset{\sim}{d} + (\underset{\sim}{b} - \underset{\sim}{c})) +$

$(\underset{\sim}{b} + (\underset{\sim}{c} - \underset{\sim}{d}))\cdot(\underset{\sim}{b} + (\underset{\sim}{c} - \underset{\sim}{d}))$

$= |\underset{\sim}{c}|^2 + 2\underset{\sim}{c}(\underset{\sim}{d} - \underset{\sim}{b}) + |\underset{\sim}{d} - \underset{\sim}{b}|^2 +$

$|\underset{\sim}{d}|^2 + 2\underset{\sim}{d}(\underset{\sim}{b} - \underset{\sim}{c}) + |\underset{\sim}{b} - \underset{\sim}{c}|^2 +$

$|\underset{\sim}{b}|^2 + 2\underset{\sim}{b}(\underset{\sim}{c} - \underset{\sim}{d}) + |\underset{\sim}{c} - \underset{\sim}{d}|^2$

$= |\underset{\sim}{b}|^2 + |\underset{\sim}{c}|^2 + |\underset{\sim}{d}|^2 + |\underset{\sim}{b} - \underset{\sim}{c}|^2 + |\underset{\sim}{d} - \underset{\sim}{b}|^2 + |\underset{\sim}{c} - \underset{\sim}{d}|^2 +$

$2(\underset{\sim}{c}\cdot\underset{\sim}{d} - \underset{\sim}{c}\cdot\underset{\sim}{b} + \underset{\sim}{d}\cdot\underset{\sim}{b} - \underset{\sim}{d}\cdot\underset{\sim}{c} + \underset{\sim}{b}\cdot\underset{\sim}{c} - \underset{\sim}{b}\cdot\underset{\sim}{d})$

$= \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{BD}\right|^2 + \left|\overrightarrow{CD}\right|^2 + 0$

$= \text{LHS}$

14. Vectors, EXT2 V1 2021 HSC 16a

i. Triangle inequality: $|x| + |y| \geq |x+y|$

♦ *Mean mark (i) 50%.*

$|x| + |y| + |z| = \left|x\underset{\sim}{i}\right| + \left|y\underset{\sim}{j}\right| + \left|z\underset{\sim}{k}\right|$

$\geq \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| + \left|z\underset{\sim}{k}\right|$

$\geq \left|x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}\right|$

$\geq 1 \quad \left(\left|\overrightarrow{OP}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}\right| = 1\right)$

ii. Using the dot product:

♦ *Mean mark (ii) 42%.*

$\underset{\sim}{a}\cdot\underset{\sim}{b} = a_1b_1 + a_2b_2 + a_3b_3$

$\underset{\sim}{a}\cdot\underset{\sim}{b} = \left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta$

$a_1b_1 + a_2b_2 + a_3b_3 = \sqrt{a_1^2 + a_2^2 + a_3^2}\cdot\sqrt{b_1^2 + b_2^2 + b_3^2}\cdot\cos\theta$

$|a_1b_1 + a_2b_2 + a_3b_3| = \sqrt{a_1^2 + a_2^2 + a_3^2}\cdot\sqrt{b_1^2 + b_2^2 + b_3^2}\cdot|\cos\theta|$

Since $-1 \leq \cos\theta \leq 1 \Rightarrow |\cos\theta| \leq 1$

$\therefore\ |a_1b_1 + a_2b_2 + a_3b_3| \leq \sqrt{a_1^2 + a_2^3 + a_3^2}\cdot\sqrt{b_1^2 + b_2^2 + b_3^2}$

iii. Using part (ii) with vectors:

♦♦♦ *Mean mark (iii) 14%.*

$\underset{\sim}{a} = \begin{pmatrix}1\\1\\1\end{pmatrix},\ \underset{\sim}{b} = \begin{pmatrix}|x|\\|y|\\|z|\end{pmatrix}$

$|\,|x| + |y| + |z|\,| \leq \sqrt{1^2 + 1^2 + 1^2}\cdot\sqrt{x^2 + y^2 + z^2}$

$|x| + |y| + |z| \leq \sqrt{3}$

15. Vectors, EXT2 V1 2022 HSC 14a

i. $\lambda\vec{u} + \mu\vec{v} = 0$

$$\lambda\vec{u} = -\mu\vec{v}$$

$$\lambda = 0 \text{ or } \vec{u} = -\left(\frac{\mu}{\lambda}\right)\vec{v} = k\vec{v} \quad (k \in \mathbb{R})$$

Since $\vec{u}$ and $\vec{v}$ are not parallel

$$\Rightarrow \lambda = \mu = 0$$

ii. $\lambda_1\vec{u} + \mu_1\vec{v} = \lambda_2\vec{u} + \mu_2\vec{v}$

$$\lambda_1\vec{u} - \lambda_2\vec{u} + \mu_1\vec{v} - \mu_2\vec{v} = \vec{0}$$

$$(\lambda_1 - \lambda_2)\vec{u} + (\mu_1 - \mu_2)\vec{v} = \vec{0}$$

Using part (i):

$(\lambda_1 - \lambda_2) = 0$ and $(\mu_1 - \mu_2) = 0$

$\therefore \lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$ ... as required

Mean mark (ii) 56%.

iii. $\overrightarrow{BL} = \lambda\overrightarrow{BC}$

$$= \lambda\left(\overrightarrow{BS} + \overrightarrow{SC}\right) \quad \ldots (1)$$

$$\overrightarrow{BL} = \overrightarrow{BS} + \mu\overrightarrow{SK}$$

$$= -\overrightarrow{SB} + \mu\left(\frac{1}{4}\overrightarrow{SB} + \frac{1}{3}\overrightarrow{SC}\right)$$

$$= -\overrightarrow{SB} + \frac{\mu}{4}\overrightarrow{SB} + \frac{\mu}{3}\overrightarrow{SC}$$

$$= \frac{\mu}{3}\overrightarrow{SC} + \left(\frac{\mu}{4} - 1\right)\overrightarrow{SB} \quad \ldots (2)$$

♦♦♦ Mean mark (iii) 25%.

Using $(1) = (2)$:

$$\lambda\overrightarrow{BS} + \lambda\overrightarrow{SC} = \frac{\mu}{3}\overrightarrow{SC} + \left(\frac{\mu}{4} - 1\right)\overrightarrow{SB}$$

$$\frac{\mu}{3} = \lambda, \quad 1 - \frac{\mu}{4} = \lambda$$

$$\frac{\mu}{3} = 1 - \frac{\mu}{4}$$

$$\frac{4\mu + 3\mu}{12} = 1$$

$$\frac{7\mu}{12} = 1$$

$$\mu = \frac{12}{7}$$

$$\lambda = \frac{\frac{12}{7}}{3} = \frac{4}{7}$$

$\therefore \overrightarrow{BL} = \frac{4}{7}\overrightarrow{BC}$ ... as required

iv. $\overrightarrow{AP} = -6\overrightarrow{AB} - 8\overrightarrow{AC}$

♦♦♦ Mean mark (iv) 23%.

If P lies on AL, $\exists k$ such that $\overrightarrow{AP} = k\overrightarrow{AL}$

$$-6\overrightarrow{AB} - 8\overrightarrow{AC} = k\overrightarrow{AL}$$

$$-6\overrightarrow{AB} - 8\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = k\left(\overrightarrow{AB} + \overrightarrow{BL}\right)$$

$$-14\overrightarrow{AB} - 8\overrightarrow{BC} = k\left(\overrightarrow{AB} + \frac{4}{7}\overrightarrow{BC}\right) \quad \text{(see part (iii))}$$

$$-14\overrightarrow{AB} - 8\overrightarrow{BC} = k\overrightarrow{AB} + \frac{4k}{7}\overrightarrow{BC}$$

$$k = -14, \quad \frac{4k}{7} = -8 \quad \Rightarrow \quad k = -14$$

$\therefore P$ lies on AL.

16. Vectors, EXT2 V1 2023 HSC 15c

♦♦♦ Mean mark 22%.

Since the curve lies on a sphere with radius 3:

$x^2 + y^2 + z^2 = 3^3$

Considering the graph $y = \cos(\pi t)\sqrt{9 - t^2}$ (as per hint)

$$\left(\cos(\pi t)\sqrt{9 - t^2}\right)^2 + \left(\sin(\pi t)\sqrt{9 - t^2}\right)^2 + t^2 = 3^2 \ \ldots (1)$$

Since z increases and x and y change signs

$\Rightarrow z = t$

In order to satisfy the equation in (1):

x, y must be one of $\pm\cos(\pi t)\sqrt{9 - t^2}$ or $\pm\sin(\pi t)\sqrt{9 - t^2}$

At $z = 0,\ t = 0,\ x = 3$ (from graph):

$\Rightarrow x = \cos(\pi t)\sqrt{9 - t^2}$

At $z = 0 + \epsilon,\ t = 0 + \epsilon,\ y < 0$ (from graph):

$\Rightarrow y = -\sin(\pi t)\sqrt{9 - t^2}$